English summary

2.3 Hindmarsh-Rose model

$$\left.\begin{aligned}\dot{x} &= c\left(x - \frac{x^3}{3} - y + z\right)\\ \dot{y} &= \frac{1}{c}\left(x^2 + dx - by + a\right)\end{aligned}\right\}$$ timeseries, Hopf and SNIPER bifurcation

3. Physiological models

3.1 Hodgkin-Huxley model

based on physical, electrical, and chemical fundamentals
(Ohm's law, Kirchhoff's junction rule, Nernst potential)

$$C_m \dot{V} = -\bar{g}_K n^4 (V - E_K) - \bar{g}_{Na} m^3 h (V - E_{Na}) - \bar{g}_L (V - E_L)$$

V: membrane potential

E_K, E_{Na}, E_L: reversal potential of K, Na, leak

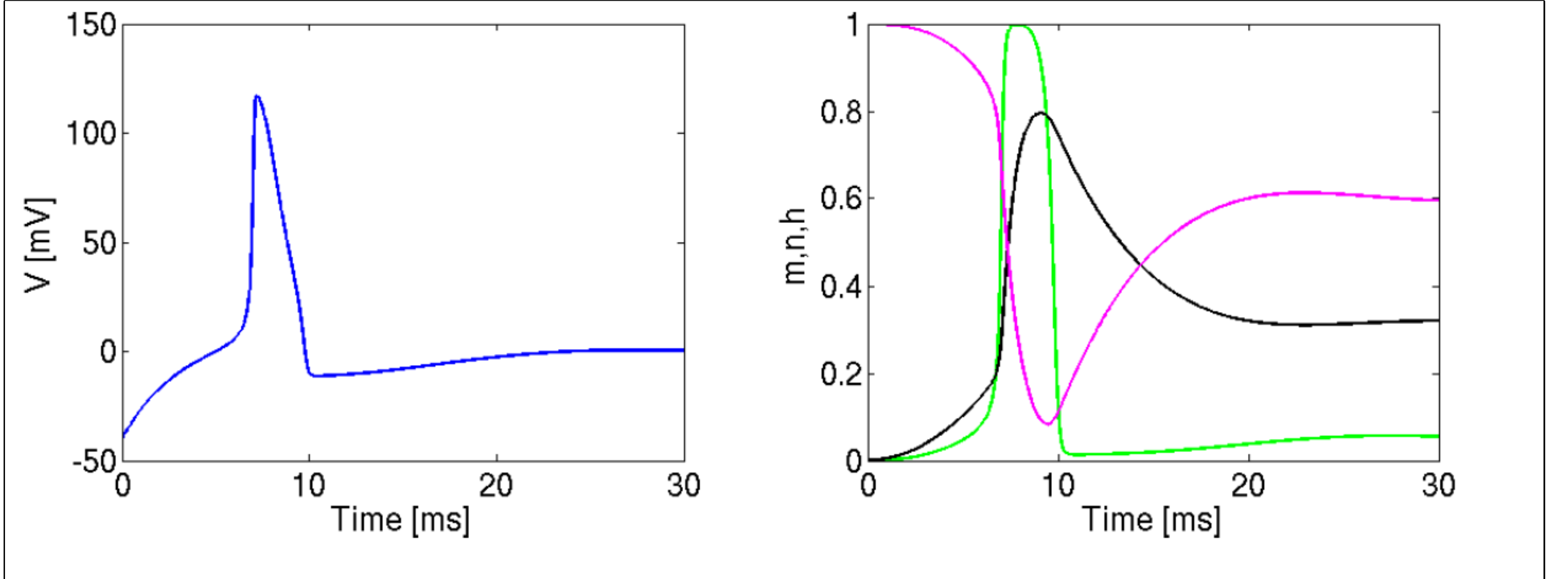
$\bar{g}_K, \bar{g}_{Na}, \bar{g}_L$: (maximum) conductances of K, Na, leak channels

$n(V)$: gating variable (K activation)
$m(V)$: " " (Na activation)
$h(V)$: " " (Na inactivation)

$$\dot{n} = \alpha_n(V)(1-n) - \beta_n(V) n$$
$$\dot{m} = \alpha_m(V)(1-m) - \beta_m(V) m$$
$$\dot{h} = \alpha_h(V)(1-h) - \beta_h(V) h$$

$$\boxed{\begin{aligned}\alpha_n(V) &= 0.01\frac{10-V}{e^{(10-V)/10}-1}\\ \beta_n(V) &= 0.125e^{-V/80}.\end{aligned}}$$

$$\boxed{\begin{aligned}\alpha_m(V) &= 0.1\frac{25-V}{e^{(25-V)/10}-1}\\ \beta_m(V) &= 4e^{-V/18}\\ \alpha_h(V) &= 0.07e^{-V/20}\\ \beta_h(V) &= \frac{1}{e^{(30-V)/10}+1}.\end{aligned}}$$

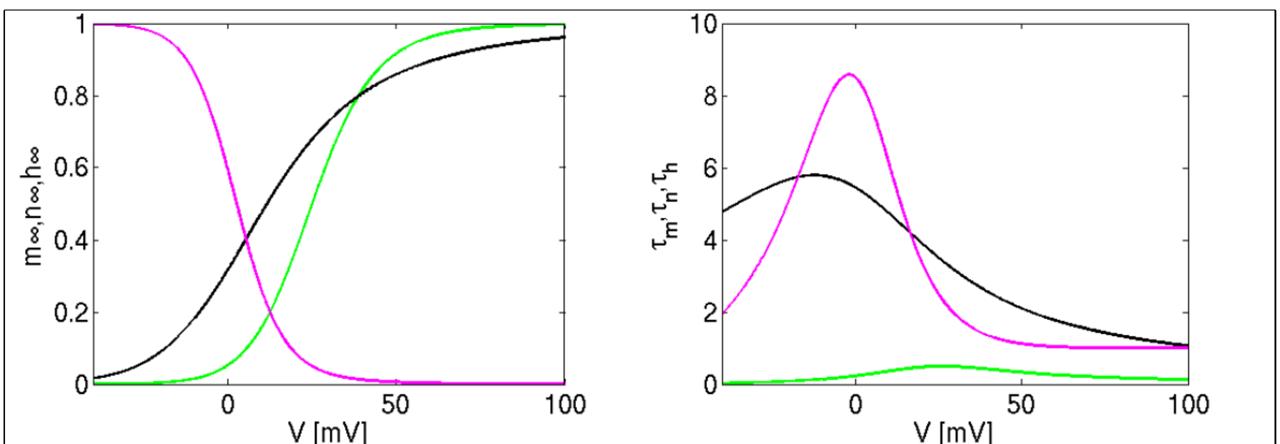


Left panel: Membrane voltage. Right panel: Activation potential, for sodium activation m (green) and inactivation h (pink) and potassium activation n (black).

3.1 Hodgkin-Huxley-Modell (Fortsetzung)

alternative Schreibweise der n, m, h-Gleichungen:

$$\tau_K = \frac{1}{\alpha_K + \beta_K}, \quad K_\infty = \frac{\alpha}{\alpha_K + \beta_K}, \quad K = n, m, h$$



Left panel:Steady state activation and inactivaion functions. Right panel: Voltage dependent time constants, for sodium activation m (green) and inactivation h (pink) and potassium activation n (black).

$$
\begin{aligned}
\tau_m(V)\frac{dm}{dt} &= -m + m_\infty(V)\\
\tau_h(V)\frac{dh}{dt} &= -h + h_\infty(V)\\
\tau_n(V)\frac{dn}{dt} &= -n + n_\infty(V)
\end{aligned}
$$

Lösung (für konstantes V): ("voltage clamp")

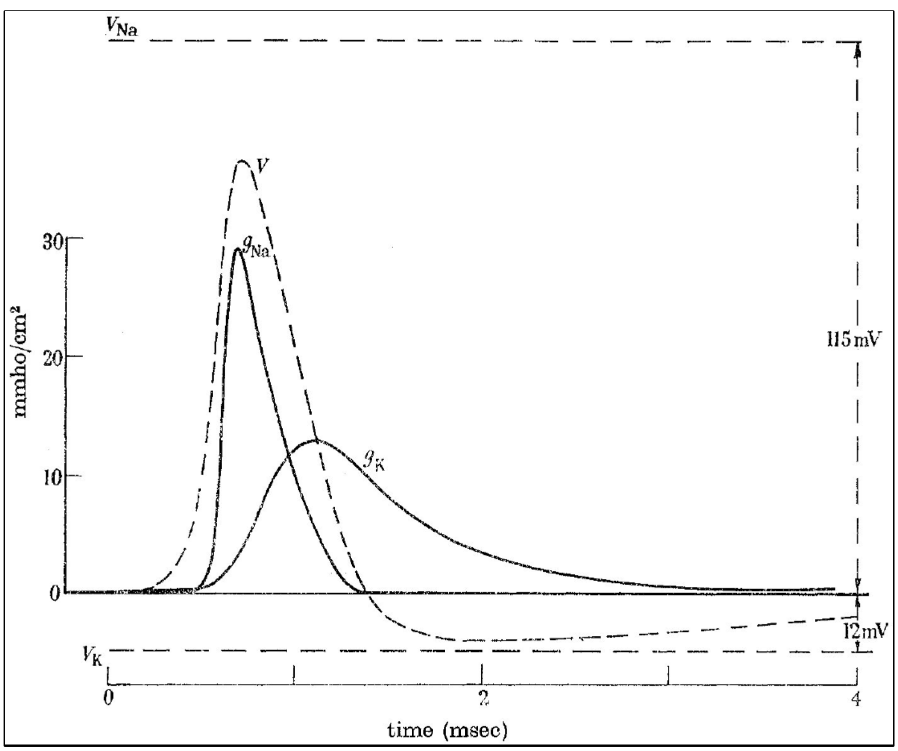
$$n(t) = n_\infty + (n_0 - n_\infty)\, e^{-t/\tau_n} = n_0 - (n_0 - n_\infty)\left(1 - e^{-t/\tau_n}\right)$$

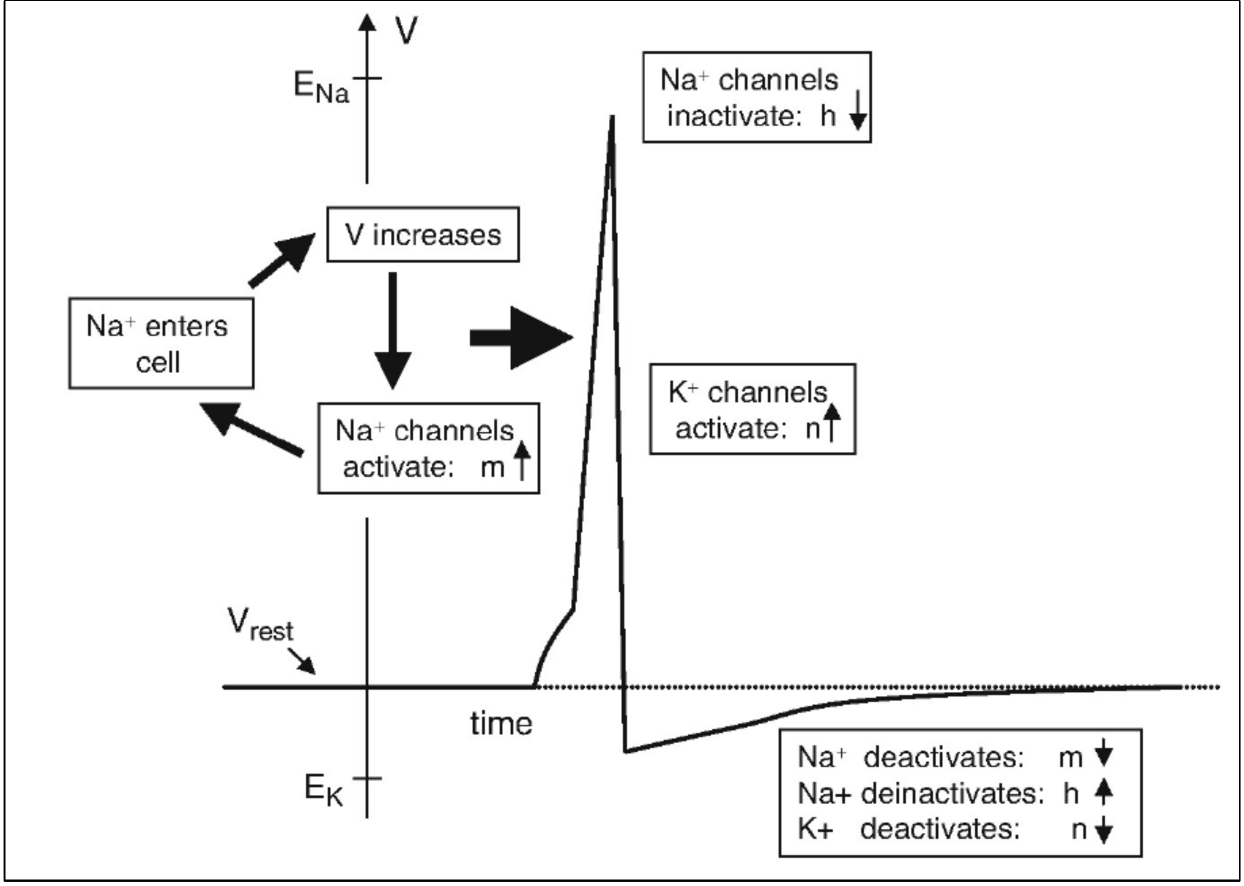
$$m(t) = m_0 - (m_0 - m_\infty)\left(1 - e^{-t/\tau_m}\right)$$

$$h(t) = h_0 - (h_0 - h_\infty)\left(1 - e^{-t/\tau_h}\right)$$

$\Rightarrow$ zeitabhängige/spannungsabhängige Leitfähigkeiten

g_K, g_{Na}, g_L

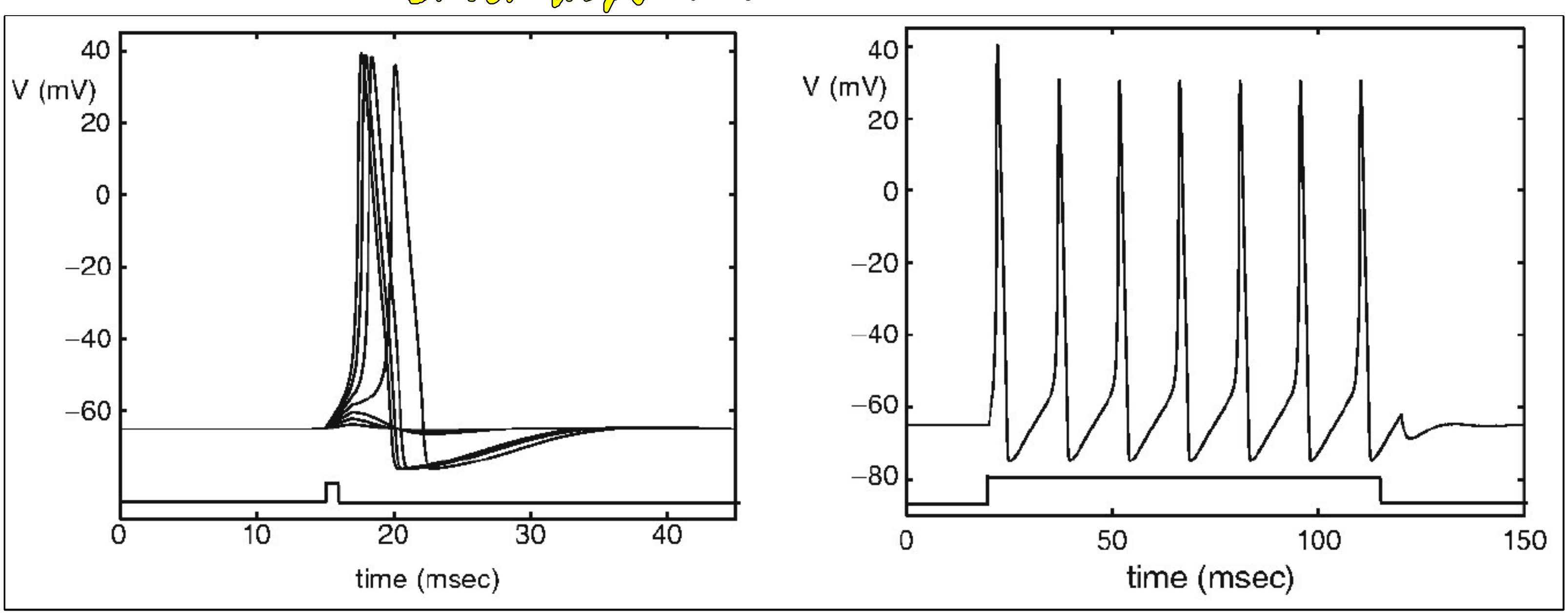




dynamische Verhalten: zusätzlicher Strom als Bifurkations-parameter I

$$C_m \dot{V} = I - I_{ion}$$

I erhöht $\Rightarrow$ Fixpunkt (Ruhezustand) wird instabil in einer Hopf-Bifurkation.



3.2 Morris-Lecar-Modell

VOLTAGE OSCILLATIONS IN THE BARNACLE GIANT MUSCLE FIBER

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Volume 35 July 1981 193–213

2-Variablen Modell:

$$C_m \dot{V} = I - g_L (V - E_L) - g_{Ca} \underbrace{m_\infty(V)}_{\text{Ca immer im Fixpunkt (Dynamik sehr schnell)}} (V - E_{Ca}) - g_k\, w(V) (V - E_k)$$

$$\tau_w(V)\, \dot{w} = w_\infty(V) - w$$

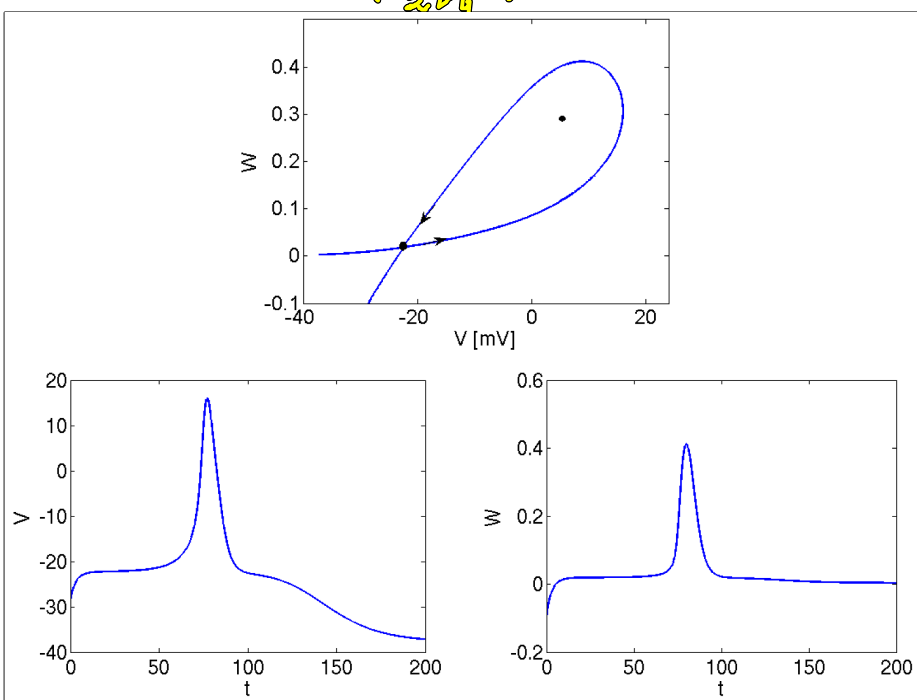
und folgende Parameter:

$$m_\infty(V) = \frac{1}{2}\left[1 + \tanh\left(\frac{V - V_1}{V_2}\right)\right] = \left[1 + \exp\left(-2\,\frac{V - V_1}{V_2}\right)\right]^{-1}$$

$$w_\infty(V) = \frac{1}{2}\left[1 + \tanh\left(\frac{V - V_3}{V_4}\right)\right] = \left[1 + \exp\left(-2\,\frac{V - V_3}{V_4}\right)\right]^{-1}$$

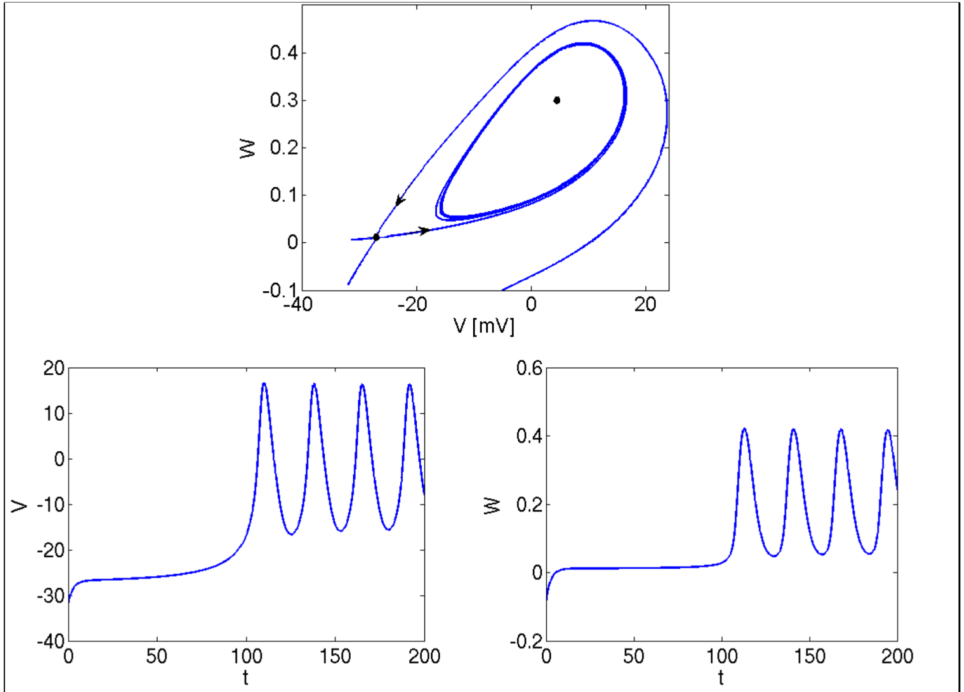
$$\tau_w(V) = \frac{1}{\cosh\left(\frac{V - V_3}{2V_4}\right)}$$

, V_1, V_2, V_3, V_4: Parameter zum Einstellen des Fixpunkts



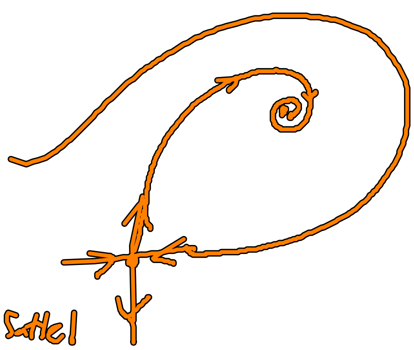
The phase portrait of the homoclinic bifurcation present in the Morris-Lecar model at $I_0 = 35$ and corresponding time series for v and ω.

homokline Bifurkation



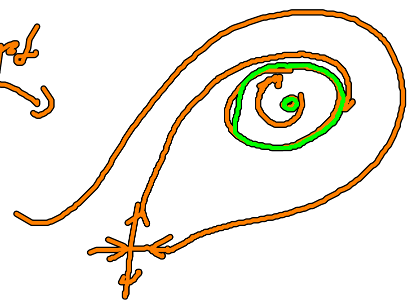
The phase portrait of Morris-Lecar at $I_0 = 39.5$ (oscillatory regime) and corresponding time series for v and ω.

Einschub: homokline Bifurkation (Nachtrag zu Kap. 7)

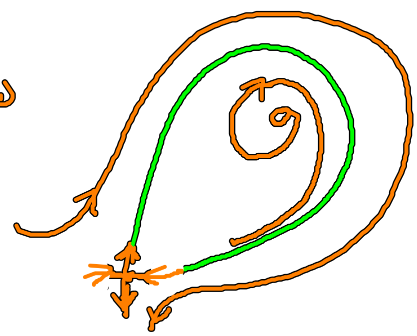


Sattel

Hopf



stabiler Grenzzyklus entsteht aus dem Fokus



Sattelpunkt wird Teil des Grenzzyklus