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VL: Prof. Dr. Harald Engel
UE: Jan Totz, M.Sc.

Projekte zur Nichtlinearen Dynamik und Musterbildung

Durchführung

Die Projekte stellen Aufgaben aus aktuellen Forschungsfeldern der Nichtlinearen Dynamik und Strukturbildung dar und können nach eigenen Vorstellungen bearbeitet werden (Numerik, Analytik, Zusammenfassung der Literatur . . .). Die in jeder Projektbeschreibung aufgeführten Punkte können als Leitfaden dienen, Sie können aber auch in Absprache mit den Betreuern eigene Ideen verfolgen.
Die Projekte sind so konzipiert, dass die Bearbeitung mit der angegebenen Literatur und dem Wissen aus der Vorlesung möglich ist. Bei einigen Projekten werden allerdings besondere Vorkenntnisse benötigt (z.B. MATHEMATICA, PYTHON, MATLAB).
Zur vollständigen Bearbeitung gehören folgende Punkte:

1. Bearbeitung des Projekts in Gruppen von maximal 2 Studierenden.

2. Präsentation der Ergebnisse in einem 15 minütigen Kurzvortrag (+ 5 Minuten Diskussion) in der vorletzten Vorlesungswoche am 14./15. Februar 2018. Wichtig ist hierbei in erster Linie die verständliche Darstellung. Beschränken Sie sich deshalb auf die zum Verständnis wesentlichen Punkte.

3. Abgabe einer schriftlichen Ausarbeitung mit vollständiger Dokumentation der Lösungswege und vollständigen Quellenangaben bis zum 16.02.2018. Auch hier steht die Verständlichkeit und übersichtliche Darstellung im Vordergrund. Der Umfang der Ausarbeitung soll 5-10 Seiten umfassen.

Während der gesamten Bearbeitungszeit stehen Ihnen die Betreuer des jeweiligen Projektes für Fragen zur Verfügung. Bitte machen Sie individuell Termine mit den Betreuenden aus.

Projekt 1: *Origin of Life from the perspective of nonlinear dynamics and pattern formation*

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The project deals with physical conditions for the emergence of life. The scope can include classical work by Oparin and Schrödinger [1, 2], work by Glansdorff/Prigogine and Ebeling [3, 4] or Manfred Eigen's theory of the hypercycle [5]. However, the focus should be on current developments as: Experiments on the origin of life in Dieter Braun's lab at LMU [6] and hydrothermal systems (black smokers) and/or physics of life-like behavior as tackled in Jeremy England's research group at MIT [7].
Tasks:

- Perform a literature review about this topic based on the given literature and the material cited therein.
- Choose a specific problem that you find interesting from current research in Braun's lab or England's group to be studied in detail.

Literature

[1] H. Rauchfuss. Chemische Evolution und der Ursprung des Lebens. Springer (2005)

[2] E. Schrödinger. What Is Life? The Physical Aspect of the Living Cell. Cambridge University Press (1944)

[3] P. Glansdorff and I. Prigogine. Thermodynamic Theory of Structure, Stability and Fluctuations. Wiley (1971)

[4] W. Ebeling. Strukturbildung bei Irreversiblen Prozessen. Eine Einführung in die Theorie dissipativer Strukturen. Teubner (1976)

[5] M. Eigen. The Hypercycle: A Principle of Natural Self-Organization. Springer (1979)

[6] C. Mast, F. M. Möller, and D. Braun. Lebendiges Nichtgleichgewicht. *Phys. J.* **10**, 29 (2013)

[7] J. L. England and G. Haran. To fold or expand—a charged question. *Proc. Natl. Acad. Sci. USA* **107**, 14519 (2010)

Projekt 2: *Mechano-chemical pattern formation*

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We know how two reacting and diffusing "morphogens" can create spontaneously a time-independent inhomogeneous morphogenetic field [1, 2]. In the seminal paper [1], Turing already realized that deformation creating mechanical stress and resulting advective flow play an important role in biological pattern formation. The aim of the project is to understand the mechano-chemical basis of morphogenesis.

Tasks:

- Perform a literature review about this topic based on the given literature and the material cited therein.
- Explore the characteristic length and time scales of mechano-chemical processes. Compare spatiotemporal patterns generated by the coupled reaction-diffusion, advection-diffusion, and reaction-advection processes as well as by viscosity and friction [3].
- For related promising directions of actual research in the field of the project compare [4] and [5, 6].
- Identify an interesting to you example for mechano-chemical pattern formation, and analyze it in depth.

Literature

[1] A. M. Turing. The chemical basis of morphogenesis. *Phil. Trans. R. Soc. Lond. B* **237**, 37 (1952)

[2] L. Wolpert. Positional information and the spatial pattern of cellular differentiation. *J. Theor. Biol.* **25**, 1 (1969)

[3] J. Howard, S. W. Grill, and J. S. Bois. Turing's next steps: The mechanochemical basis of morphogenesis. *Nat. Rev. Mol. Cell Biol.* **12**, 392 (2011)

[4] V. V. Yashin and A. C. Balazs. Pattern Formation and Shape Changes in Self-Oscillating Polymer Gels. *Science* **314**, 798 (2006)

[5] A. V. Panfilov, R. H. Keldermann, and M. P. Nash. Self-Organized Pacemakers in a Coupled Reaction-Diffusion-Mechanics System. *Phys. Rev. Lett.* **95**, 258104 (2005)

[6] A. V. Panfilov, R. H. Keldermann, and M. P. Nash. Drift and breakup of spiral waves in reaction–diffusion–mechanics systems. *Proc. Natl. Acad. Sci. USA* **104**, 7922 (2007)

Projekt 3: *Applications of catastrophe theory in physics and biology*

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Grounded in the work of Poincaré, Pontriagin, Thom, Smale and others, catastrophe theory attempts to study discontinuous changes in a system's state occurring under variations of the parameters that appear in the dynamical equations of the system. The focus of the project is rather on applications [1, 2] that illustrate the distinctive nature catastrophe theory than on its mathematical foundation [3]. The goal is to understand the ideas behind the theory [4] well enough to follow the arguments in papers in which it is used and, if the occasion arises, to use it themselves.
Tasks:

- Perform a literature review about this topic based on the given literature and the material cited therein.
- Select one or two specific applications from the areas
  - caustics in optics (diffraction integrals) [2, 5, 6]
  - buckling instabilities in elasticity [2, 7]
  - morphogenesis [3, 8]

  to be elaborated and presented in detail.
- Do work

Literature

[1] M. Golubitsky. An Introduction to Catastrophe Theory and its Applications. *SIAM Rev.* **20**, 352 (1978)

[2] T. Poston, T. Poston, and Mathematics. Catastrophe Theory and Its Applications. Dover Books on Mathematics (1997)

[3] R. Thom. Structural Stability and Morphogenesis. Perseus Books Group (1975)

[4] E. C. Zeeman. Catastrophe Theory. *Sci. Am.* **234**, 65 (1976)

[5] M. V. Berry and C. Upstill. Catastrophe Optics: Morphologies of Caustics and Their Diffraction Patterns. In E. Wolf, ed., "Progress in Optics", volume 18, 257–346. Elsevier (1980)

[6] D. van Straten. Geraden, Kurven und Kuspen (2008)

[7] J. M. T. Thompson. Instabilities and Catastrophes in Science and Engineering. Wiley (1981)

[8] R. Thom. Topological models in biology. *Topology* **8**, 313 (1969)

Projekt 4: *Turing patterns on networks*

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Turing patterns [1] in a spatially continuous system are thought to play a role in vividly colored fish skin and animal furs as well as controlling growth during early developmental stages of organisms [2, 3]. Apart from biological relevance, Turing patterns were also produced in controlled chemical experiments [4, 5, 6]. They are the result of a symmetry-breaking bifurcation, which leaves the homogeneous steady state unstable to even infinitesimal perturbations that feature certain critical spatial wavelengths k. Due to the lack of spatial continuity, Turing patterns on networks of discrete nodes are more difficult to recognize visually. Recently, however, Nakao and Mikhailov succeeded in translating the mathematical analysis from continuous media to discrete nodes and were able to identify Turing patterns on networks as well [7].

Tasks:

- Perform a literature review about this topic based on the given literature and the material cited therein.
- Prepare a comparative review of the Turing mechanism in continuous systems and discrete networks. How is the coupling on the network incorporated into the linear stability analysis? What are the conditions for Turing patterns?
- Analyze and simulate the Mimurra-Murray model used in [7].
- Simulate the Turing pattern on a large random network of $N = 1000$ nodes as in [7]. Does the resulting pattern coincide with the critical mode?
- What would be required to experimentally verify a Turing pattern on a network?

Literature

[1] A. M. Turing. The chemical basis of morphogenesis. *Phil. Trans. R. Soc. Lond. B* **237**, 37 (1952)

[2] J. D. Murray. Mathematical Biology II: Spatial Models and Biomedical Applications. Springer (2003)

[3] A. D. Economou, A. Ohazama, T. Porntaveetus, P. T. Sharpe, S. Kondo, M. A. Basson, A. Gritli-Linde, M. T. Cobourne, and J. B. A. Green. Periodic stripe formation by a Turing mechanism operating at growth zones in the mammalian palate. *Nat. Genet.* **44**, 348 (2012)

[4] V. Castets, E. Dulos, J. Boissonade, and P. De Kepper. Experimental evidence of a sustained standing Turing-type nonequilibrium chemical pattern. *Phys. Rev. Lett.* **64**, 2953 (1990)

[5] V. Vanag and I. Epstein. Pattern Formation in a Tunable Medium: The Belousov-Zhabotinsky Reaction in an Aerosol OT Microemulsion. *Phys. Rev. Lett.* **87** (2001)

[6] J. Horváth, I. Szalai, and P. D. Kepper. An Experimental Design Method Leading to Chemical Turing Patterns. *Science* **324**, 772 (2009)

[7] H. Nakao and A. S. Mikhailov. Turing patterns in network-organized activator-inhibitor systems. *Nat. Phys.* **6**, 544 (2010)

Projekt 5: *Data-driven model discovery*

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An alternative to deriving underlying equations of real-world systems from first principles is to employ machine learning on experimentally measured time series [1]. A recently developed improvement exploits the sparse structure of governing equations in the space of all possible functions [2, 3, 4, 5]. Starting from an initial set of nonlinear candidate functions, the algorithm only retains those candidates, which constitute a minimal model for the measured dataset.

Tasks:

- Perform a literature review about this topic based on the given literature and the material cited therein.
- Compare the different data-driven approaches by Lipson et al. [1], Brunton et al [2] and Kevrekidis et al [6].
- Employ the algorithm presented in [4] to identify the dynamical system behind a numerical simulation of the Stuart-Landau oscillator, the Oregonator model [7] and the Lorenz model.
- What are the difficulties in identifying the dynamical system of a traveling pulse in the Oregonator model?

Literature

[1] M. Schmidt and H. Lipson. Distilling Free-Form Natural Laws from Experimental Data. *Science* **324**, 81 (2009)

[2] S. L. Brunton, J. L. Proctor, and J. N. Kutz. Discovering governing equations from data by sparse identification of nonlinear dynamical systems. *Proc. Natl. Acad. Sci. USA* **113**, 3932 (2016)

[3] N. M. Mangan, S. L. Brunton, J. L. Proctor, and J. N. Kutz. Inferring Biological Networks by Sparse Identification of Nonlinear Dynamics. *IEEE Trans. Mol. Biol. Multi-Scale Commun.* **2**, 52 (2016)

[4] S. H. Rudy, S. L. Brunton, J. L. Proctor, and J. N. Kutz. Data-driven discovery of partial differential equations. *Sci. Adv.* **3**, e1602614 (2017)

[5] H. Schaeffer. Learning partial differential equations via data discovery and sparse optimization. *Proc. R. Soc. A* **473**, 20160446 (2017)

[6] I. G. Kevrekidis, C. W. Gear, J. M. Hyman, P. G. Kevrekidis, O. Runborg, and C. Theodoropoulos. Equation-Free, Coarse-Grained Multiscale Computation: Enabling Mocroscopic Simulators to Perform System-Level Analysis. *Commun. Math. Sci.* **1**, 715 (2003)

[7] J. F. Totz. Synchronization and Waves in Confined Complex Active Media. Phd, TU Berlin, Berlin (2017)

Projekt 6: *Bump states in arrays of chemical oscillators*

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Localized regions of elevated brain activity, which were named bump states, are thought to play an important role in various cognitive processes, such as short-term memory and the head direction system [1, 2, 3]. Their emergence can be investigated in a reduced neural network, consisting of a ring of nonlocally coupled excitable elements. Apart from numerical simulations, the chemical Belousov-Zhabotinsky reaction offers a possibility to examine self-organized patterns in an excitable continuous medium similar to neuronal tissue. Recent advances [4] in confining the reaction at discrete locations, allow for creating a large network of light-coupled units that permit well-controlled experiments on self-organized patterns on discrete networks, such as the bump state.

Tasks:

- Perform a literature review about this topic based on the given literature and the material cited therein.
- What are the conditions for a bump state to arise? What is the difference between a bump state and a chimera state [5]?
- Reproduce the simulation of the bump state with the theta neuron model in [2].
- Find suitable model and coupling parameters for bump states to occur in a nonlocally-coupled array of chemical oscillators (Oregonator model [6]).

Literature

[1] C. R. Laing and C. C. Chow. Stationary Bumps in Networks of Spiking Neurons. *Neural Comput.* **13**, 1473 (2001)

[2] C. R. Laing. Derivation of a neural field model from a network of theta neurons. *Phys. Rev. E* **90**, 010901 (2014)

[3] C. R. Laing. Bumps in Small-World Networks. *Front. Comput. Neurosci.* 53 (2016)

[4] J. F. Totz, J. Rode, M. R. Tinsley, K. Showalter, and H. Engel. Spiral wave chimera states in large populations of coupled chemical oscillators. *Nat. Phys.* 1 (2017)

[5] M. J. Panaggio and D. M. Abrams. Chimera states: Coexistence of coherence and incoherence in networks of coupled oscillators. *Nonlinearity* **28**, R67 (2015)

[6] J. F. Totz. Synchronization and Waves in Confined Complex Active Media. Phd, TU Berlin, Berlin (2017)