



Lecture 28



Partial synchrony in complex networks: interplay of structure, noise, and delay

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Winter Term 2019/20 “Non-equilibrium Statistical Physics”

synchrony

Is **synchrony** always good?

Is synchrony always good?

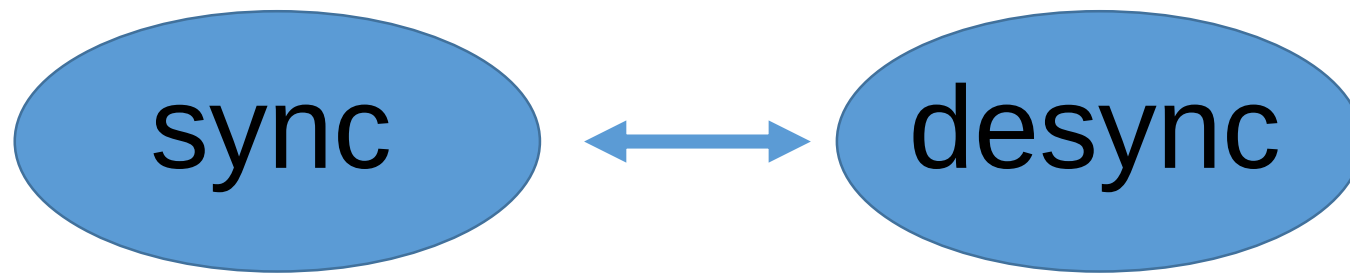


- neural networks

- power grid networks

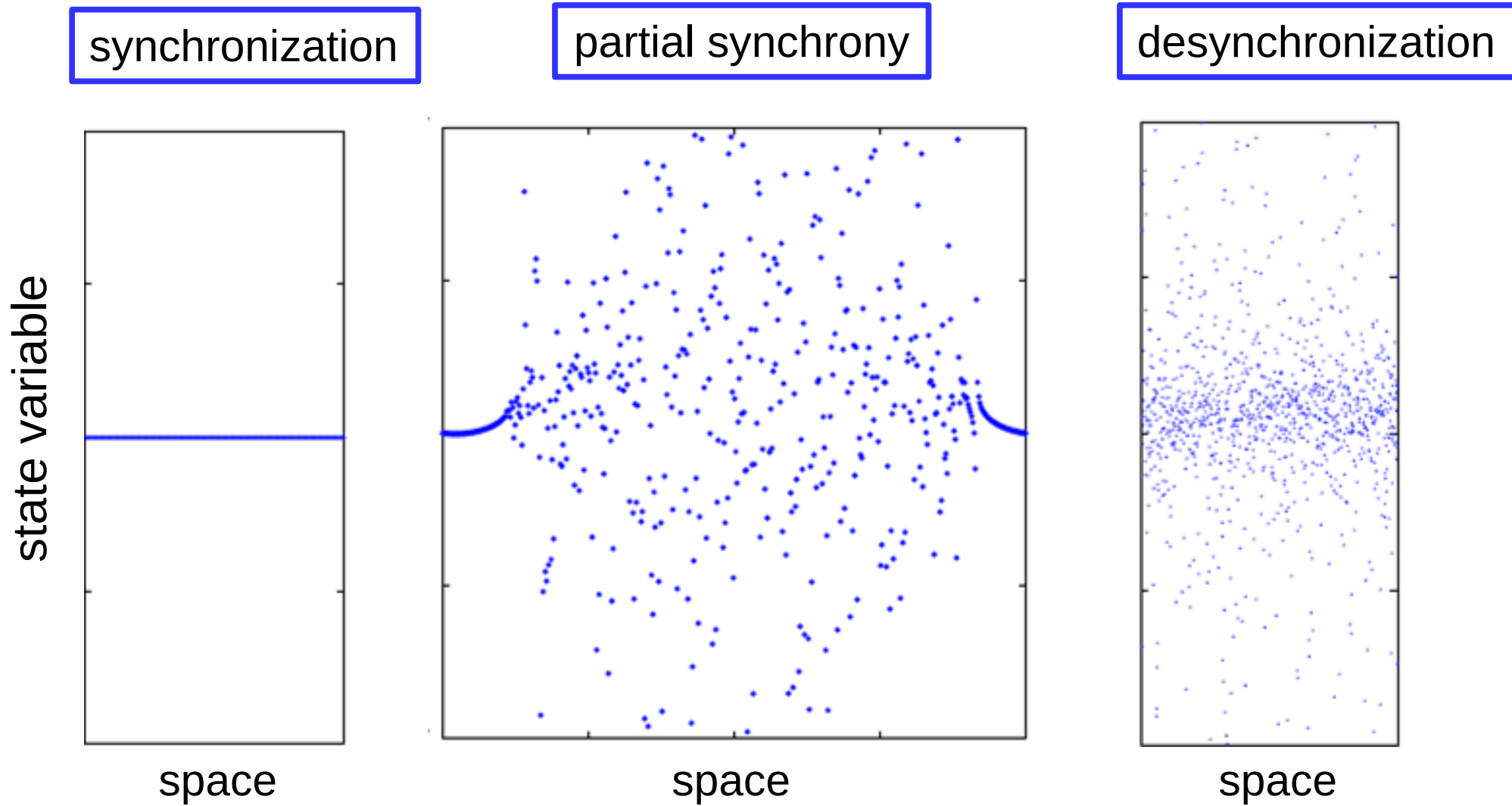


Transitions



partial synchronization patterns

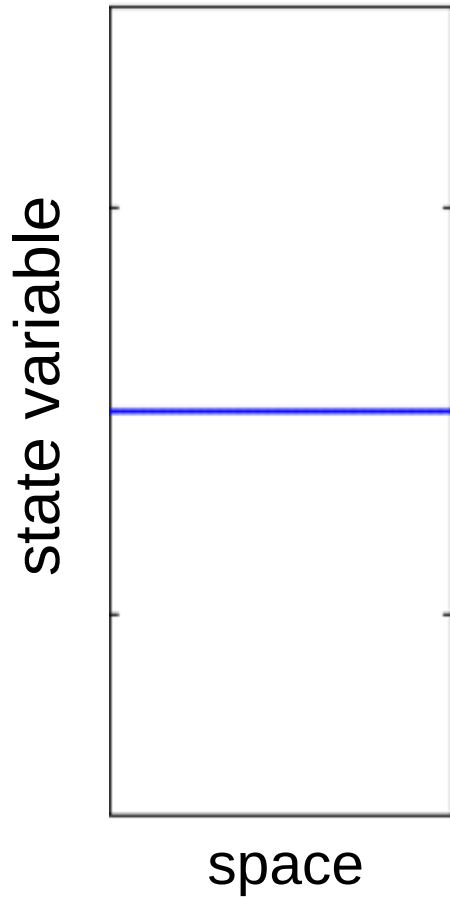
What is partial synchrony?



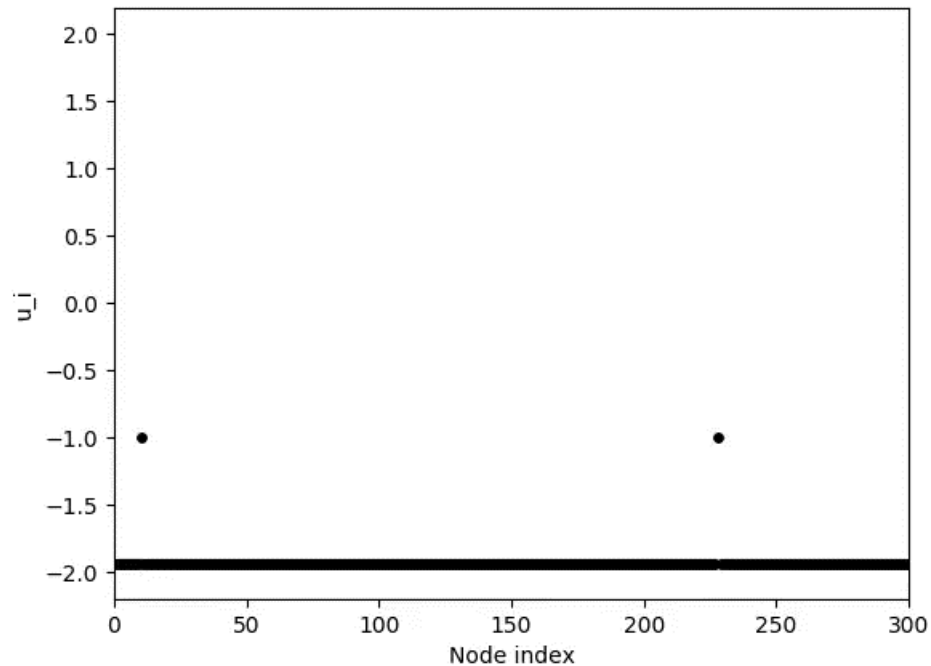
Chimera states

What is partial synchrony?

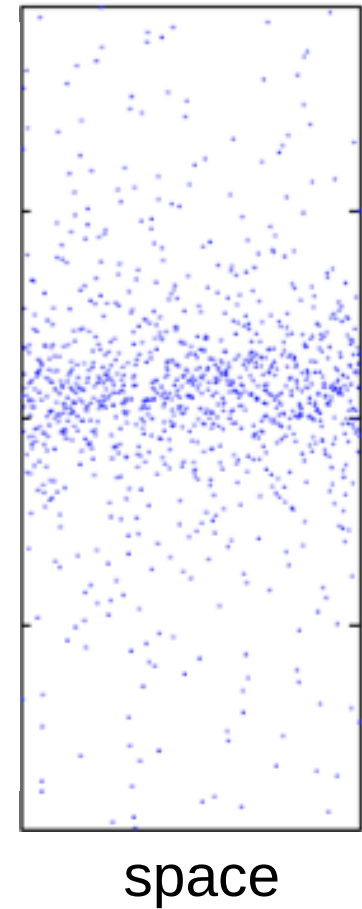
synchronization



partial synchrony



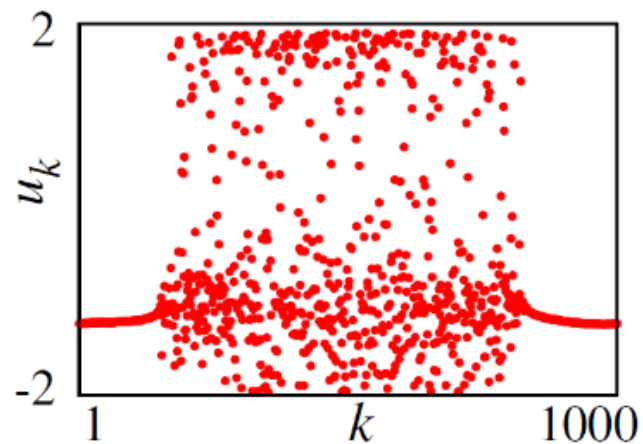
desynchronization



Solitary states

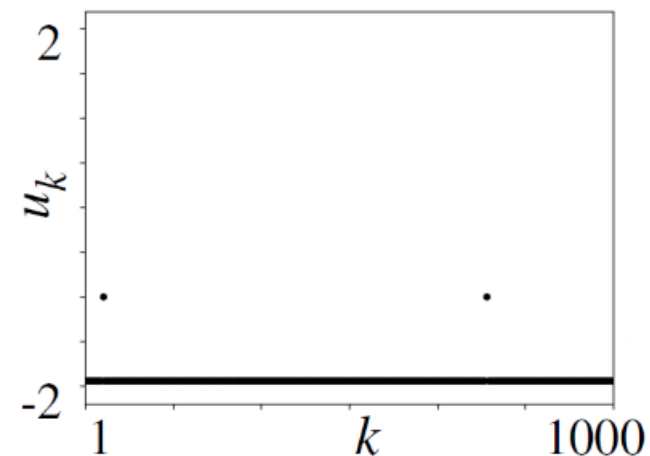
What is the difference?

chimera states



Localized in space
incoherent domain

solitary states



Randomly distributed
solitary nodes

Structure, delay and noise

Dynamics and control

Network
topology

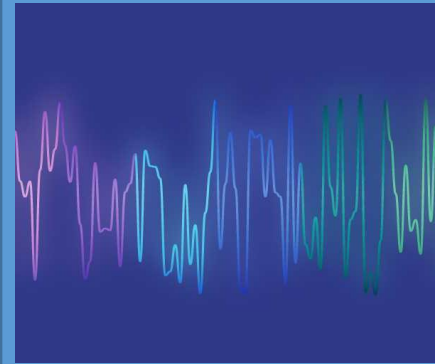


Time
delay

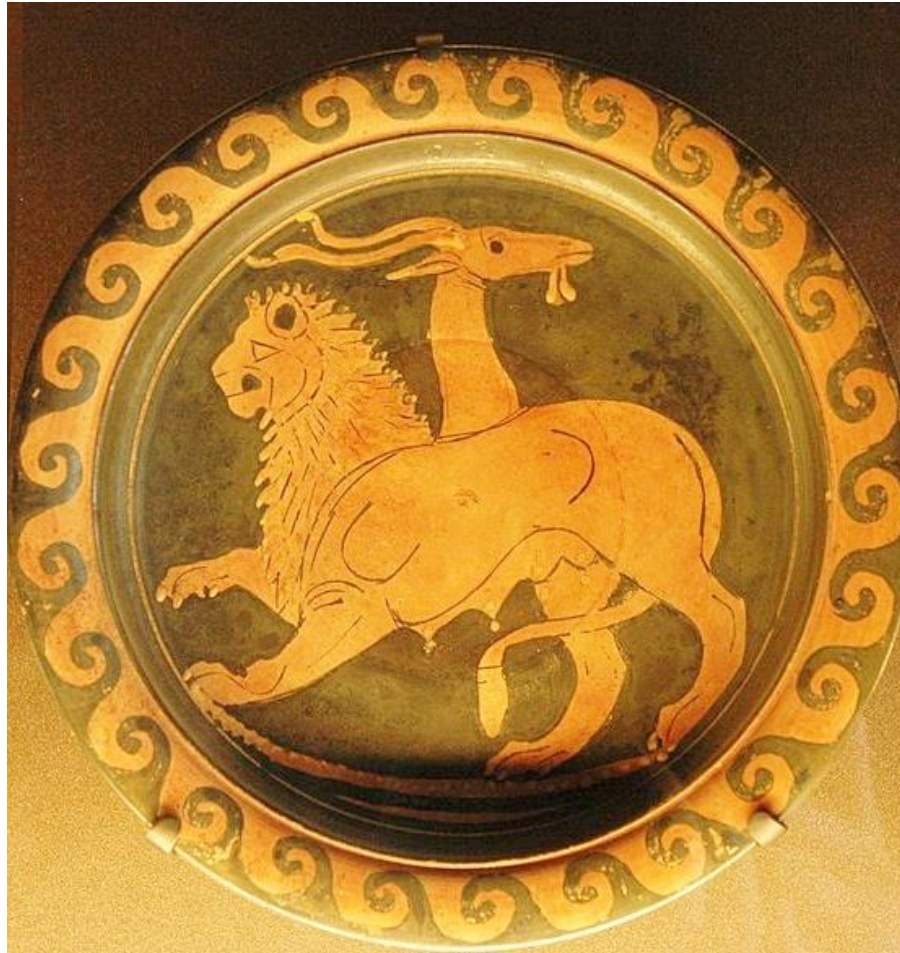
τ



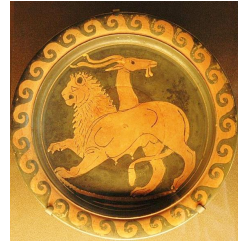
Noise



Partial sync pattern: chimera state

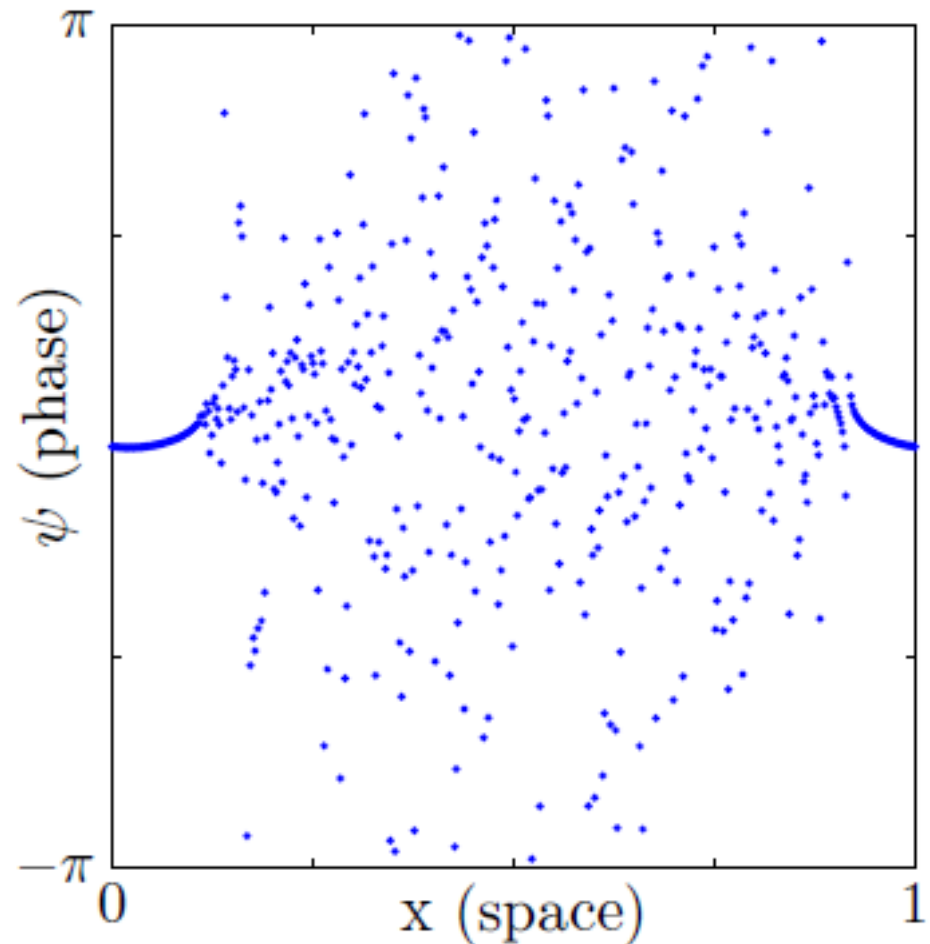


What is a chimera state?



Spatial coexistence of **coherent/synchronized** and **incoherent/desynchronized** domains in a dynamical network

- discovered by Kuramoto and Battogtokh in 2002
- named ***chimera states*** by Abrams and Strogatz in 2004
- identical elements, symmetric topology



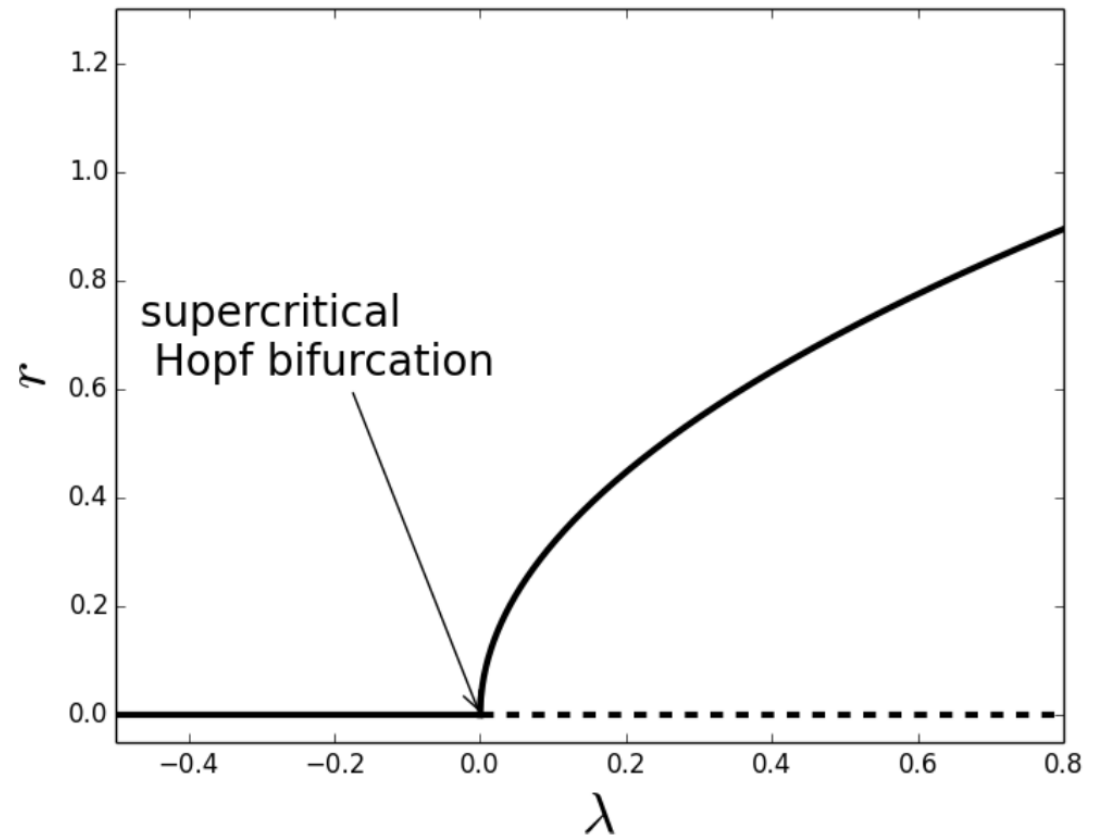
Amplitude chimeras and chimera death

Model: Stuart-Landau oscillator

Single node dynamics

$$\dot{z} = f(z) \equiv (\lambda + i\omega - |z|^2)z$$

$$z(t) = x(t) + iy(t)$$



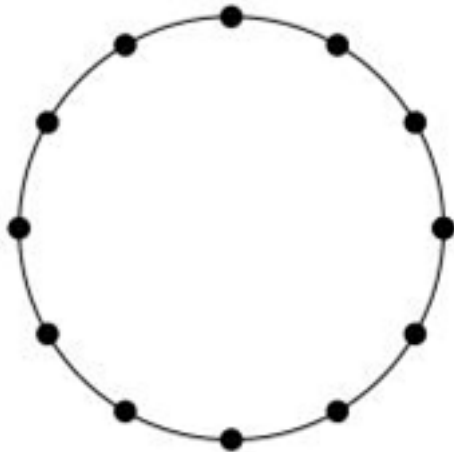
Model: Stuart-Landau oscillator

Single node dynamics

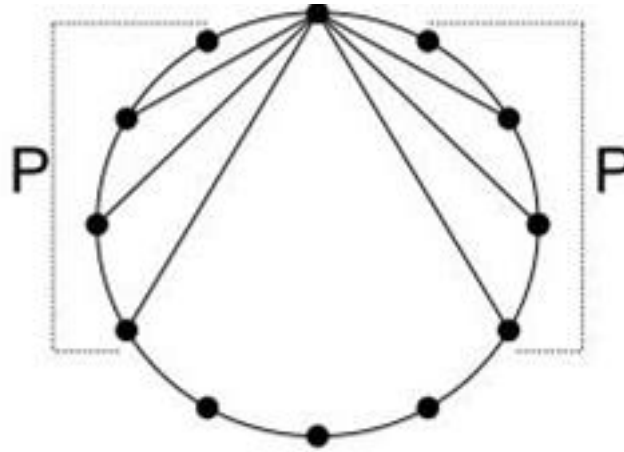
$$\dot{z} = f(z) \equiv (\lambda + i\omega - |z|^2)z$$

$$z(t) = x(t) + iy(t)$$

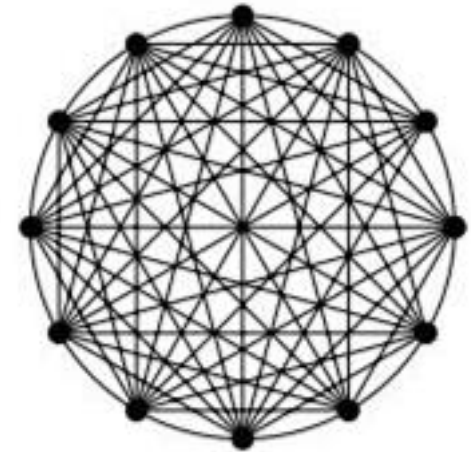
How to couple the oscillators?



local



nonlocal



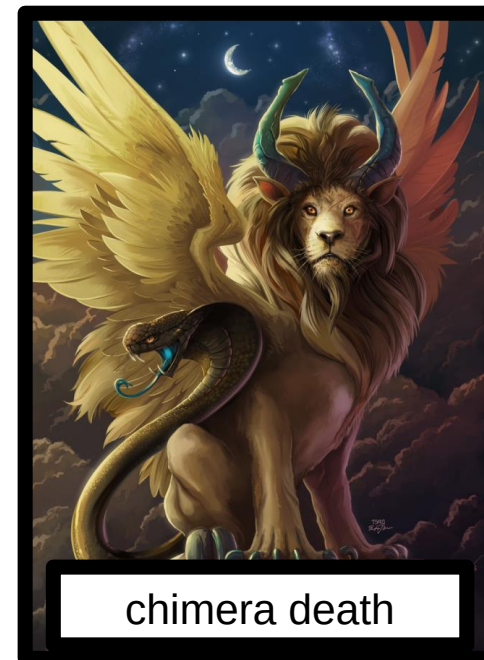
global

Model: network of **coupled** Stuart-Landau oscillators

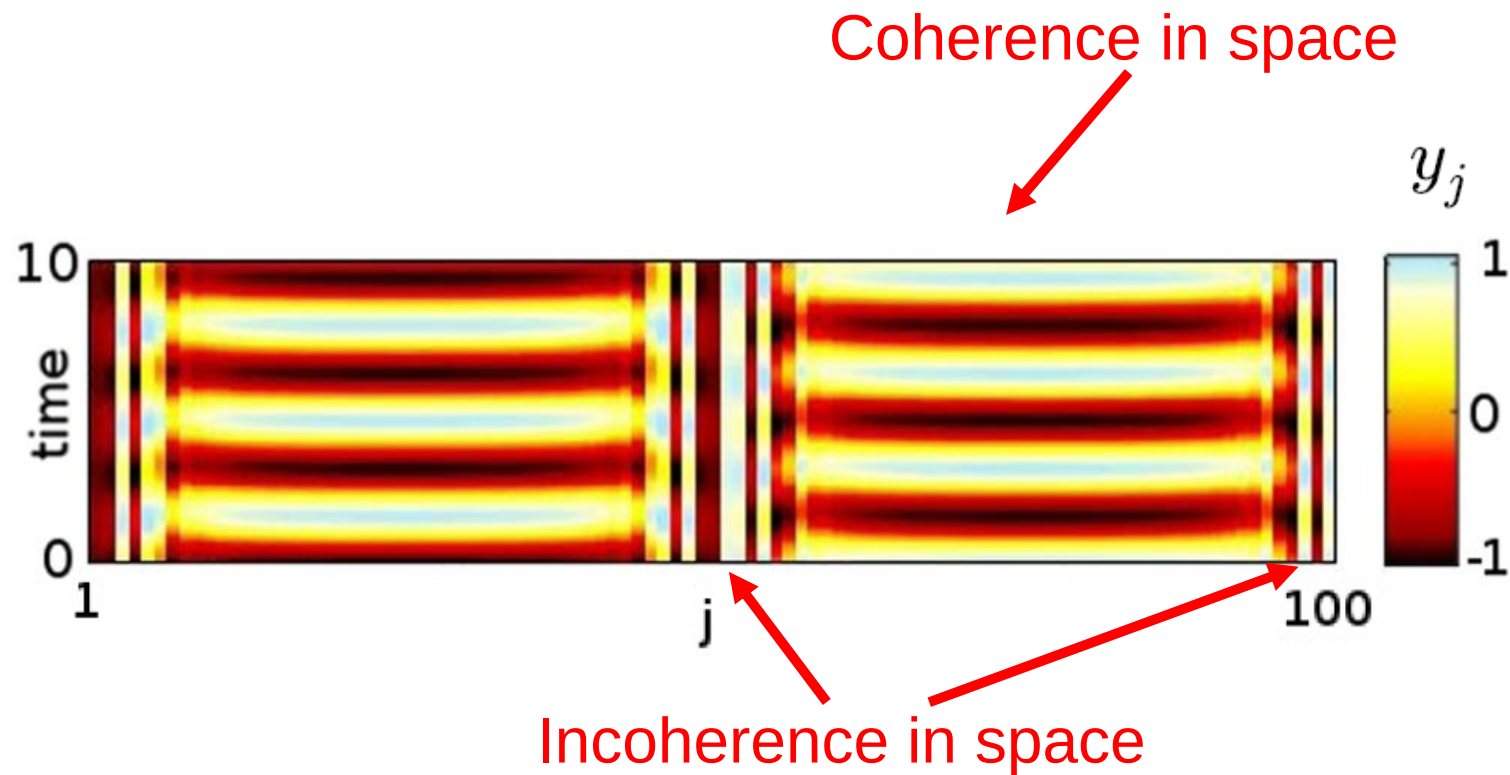


$$\dot{z}_j = f(z_j) + \frac{\sigma}{2P} \sum_{k=j-P}^{j+P} (\text{Re } z_k - \text{Re } z_j)$$

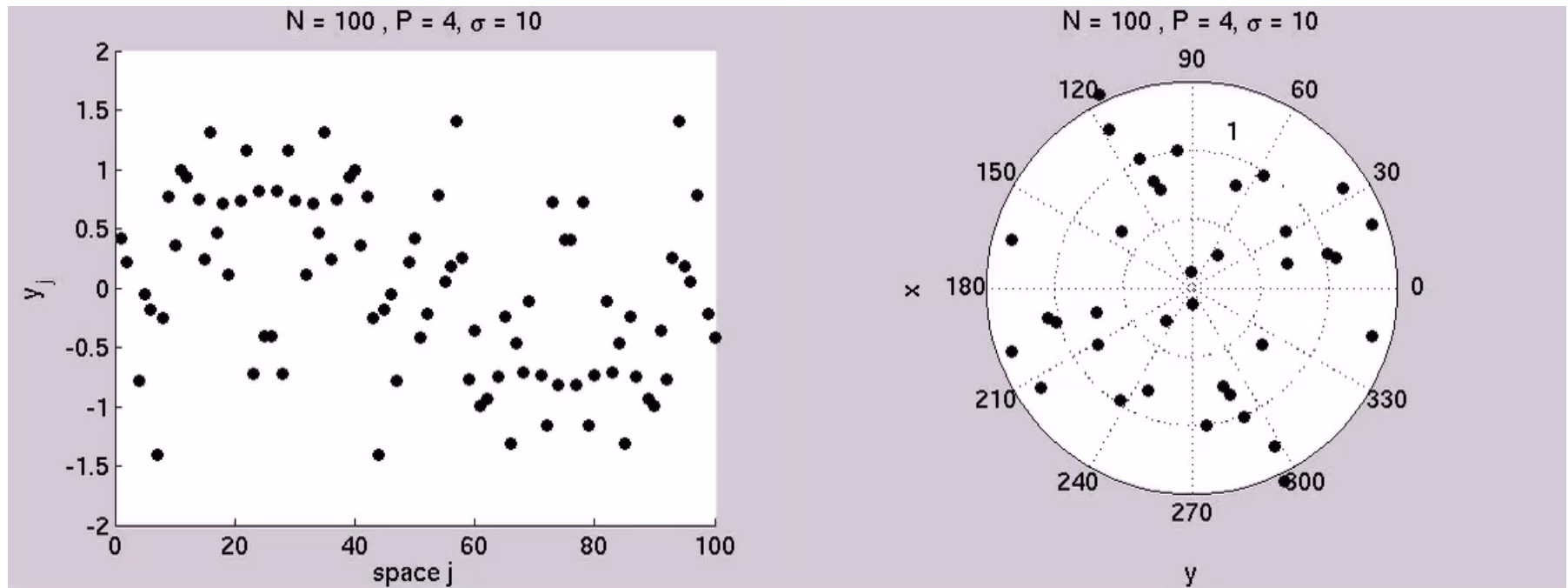
← Symmetry-breaking coupling



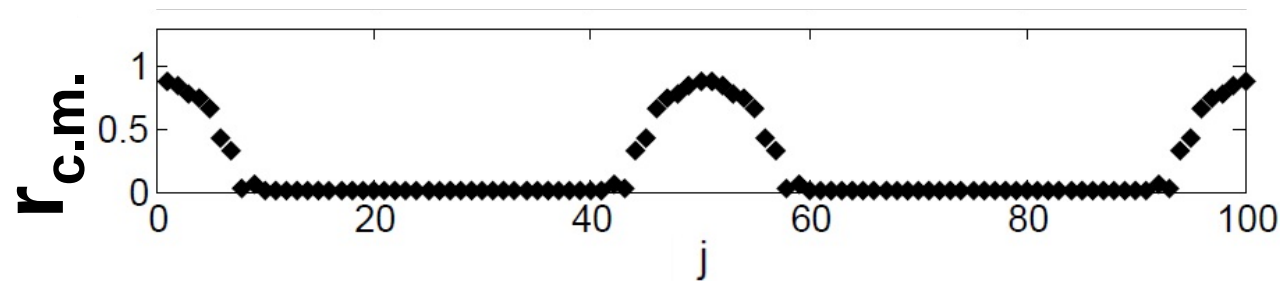
Amplitude chimera



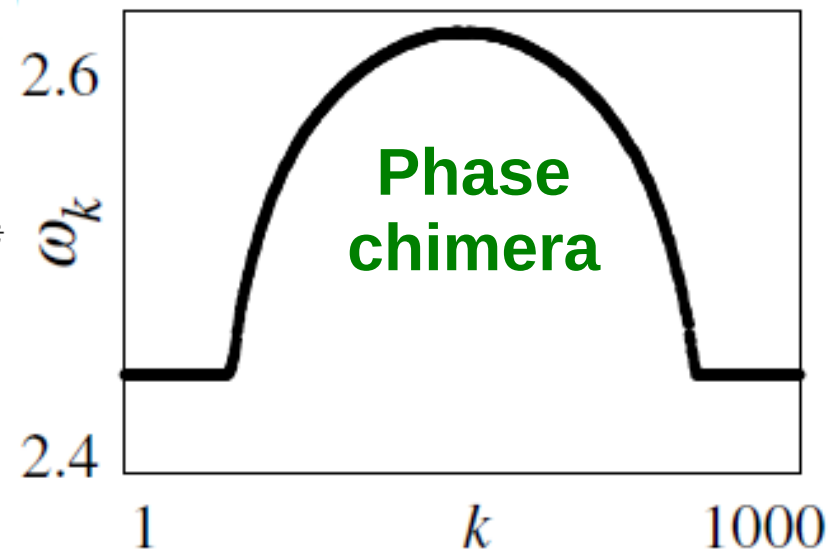
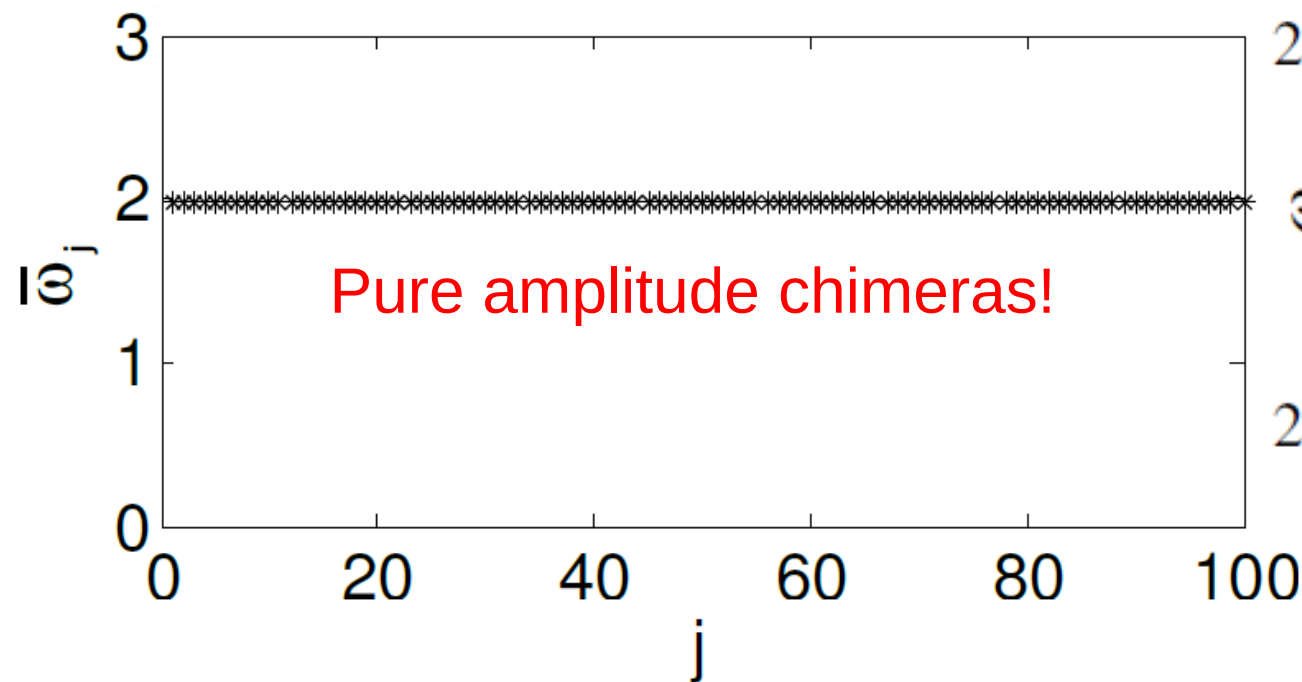
Amplitude chimera



Amplitude chimera



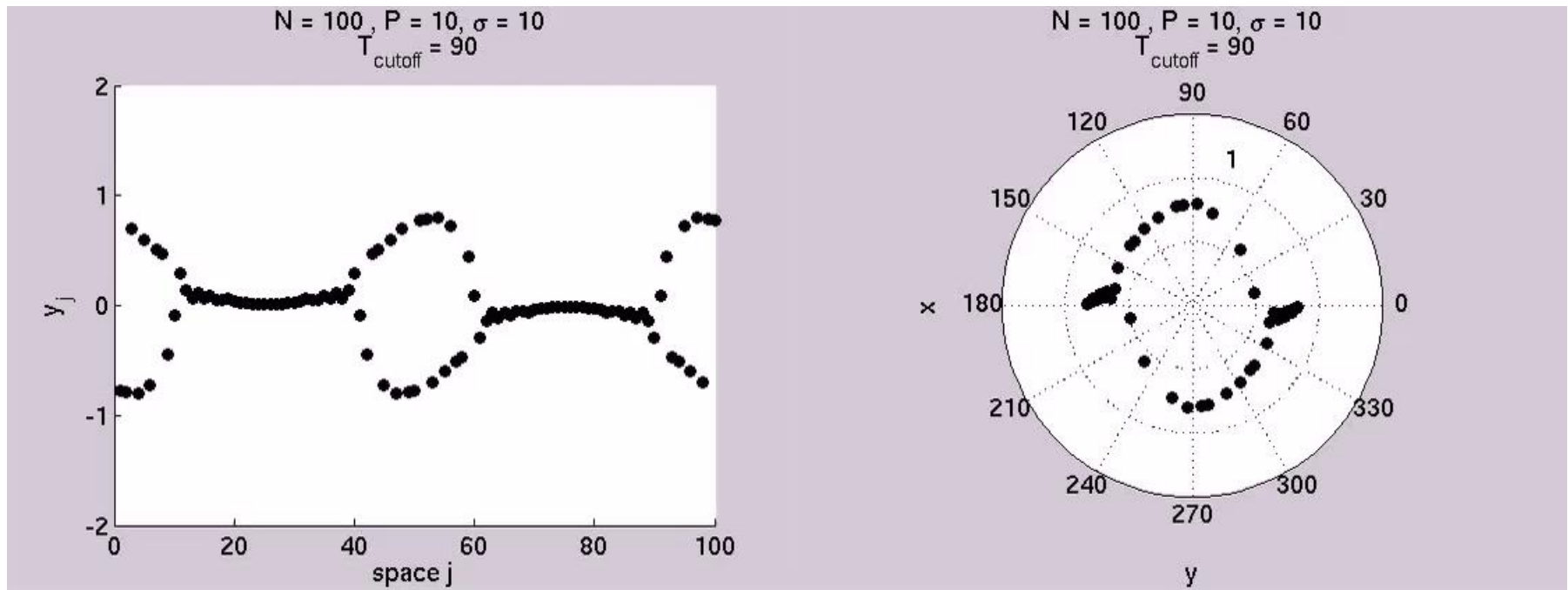
$r_{c.m.}$ is the shift of the center of mass from the origin



Amplitude chimera

	Amplitude	Phase
◦ Classical chimeras Kuramoto and Battogtokh (2002), Abrams and Strogatz (2004)	no amplitude dynamics	
◦ Amplitude-mediated chimeras Sethia and Sen (2014)		
◦ Amplitude chimeras Zakharova, Kapeller, Schöll (2014)		

Amplitude chimera



Classical chimeras:

the dynamics is **chaotic** in time

M. Wolfrum and O. Omel'chenko, Chimera states are chaotic transients, *Phys Rev E* 84, 015201 (2011)

Amplitude chimeras:

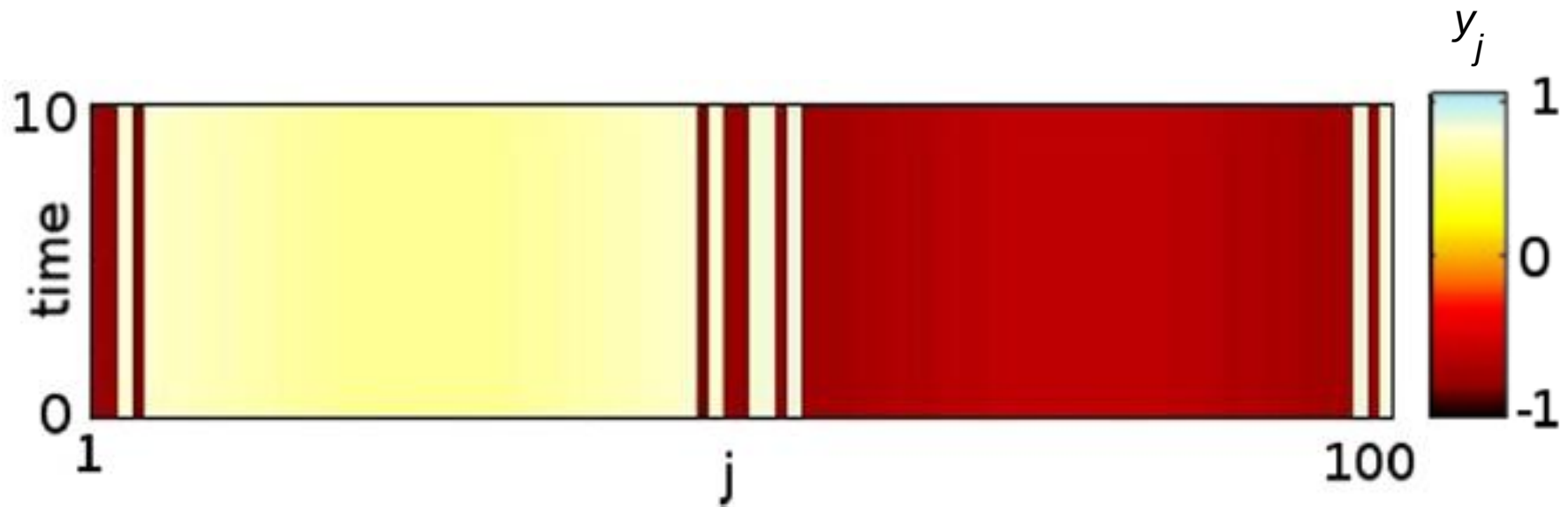
the dynamics is **periodic** in time!

Amplitude chimeras are long-living transients!

Chimera death

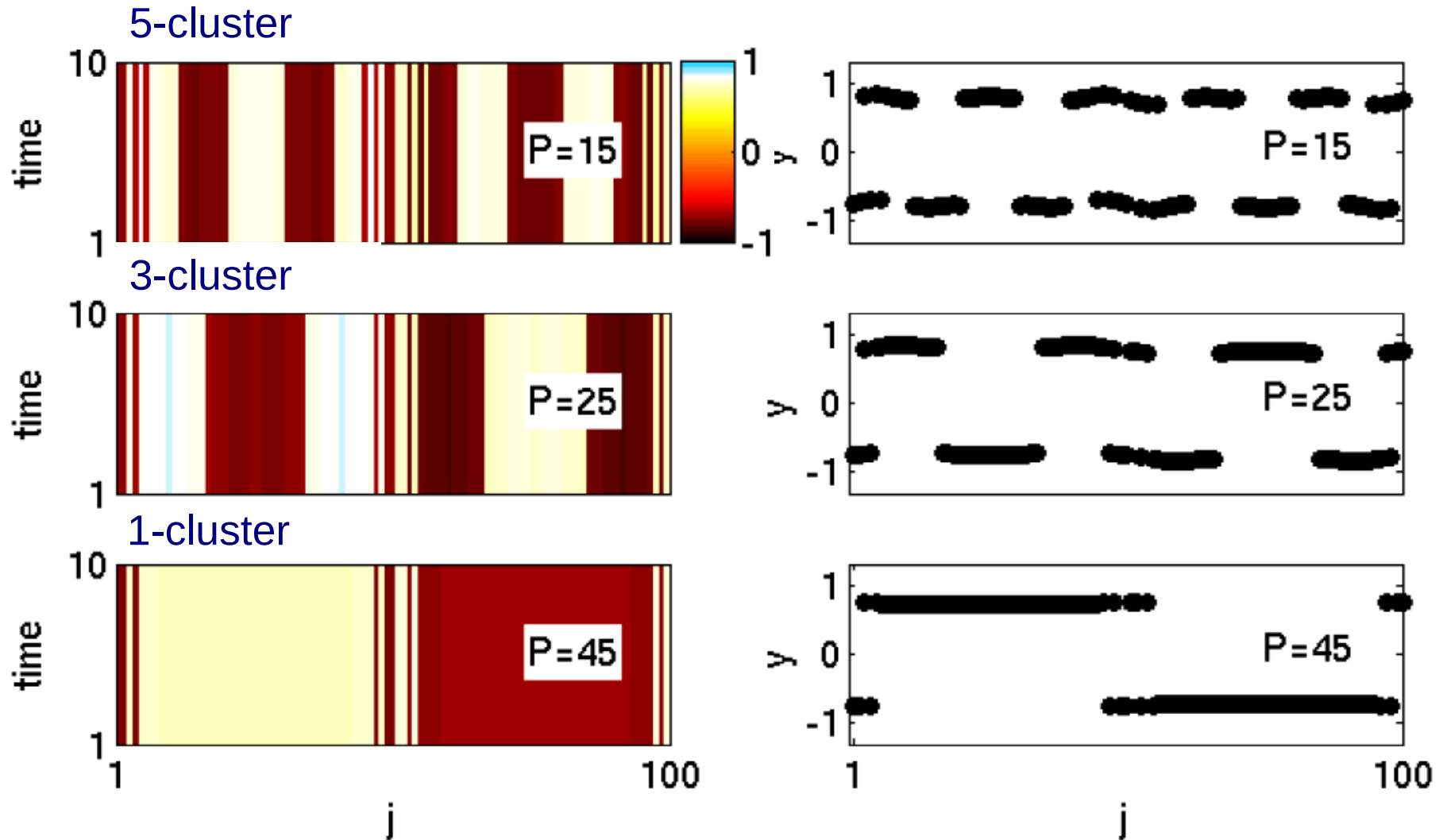
Chimera death

coexisting domains of spatially coherent
and spatially incoherent oscillation death

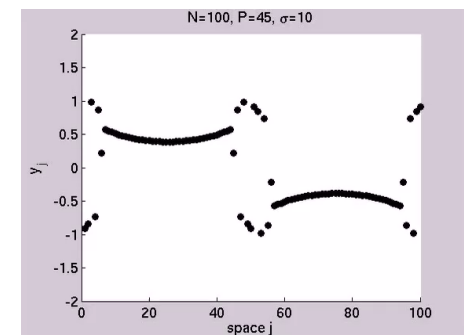
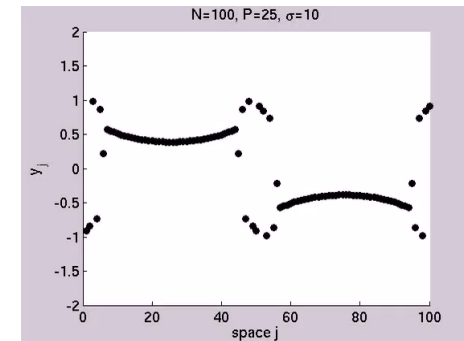
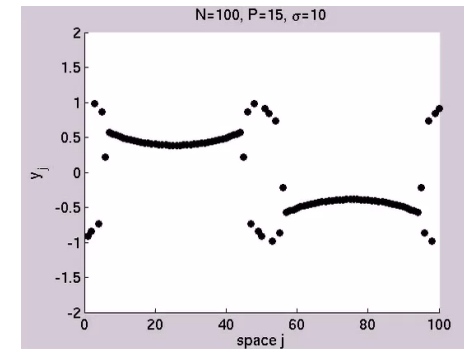
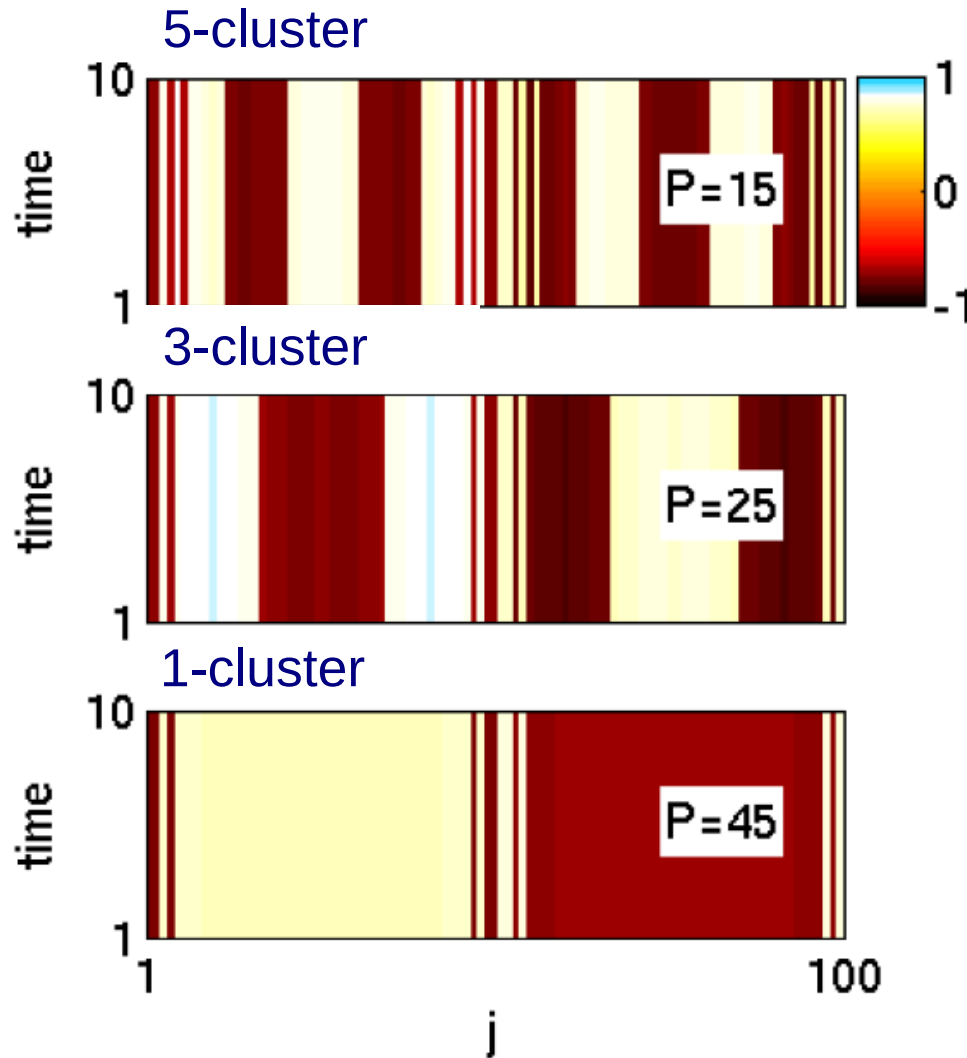


Chimera of steady states!

Chimera death: formation of clusters

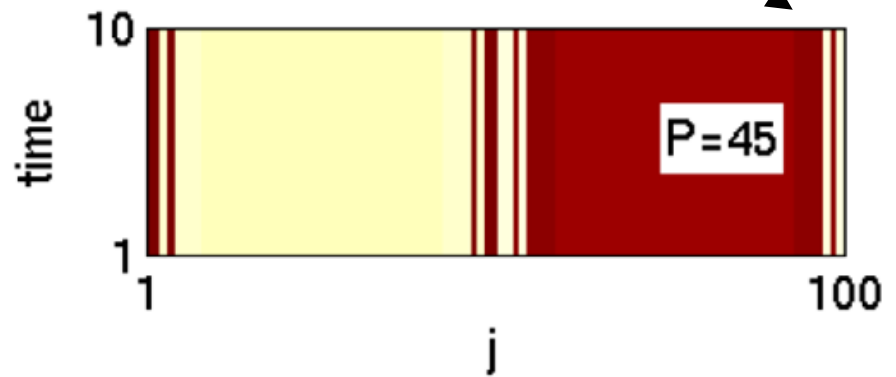
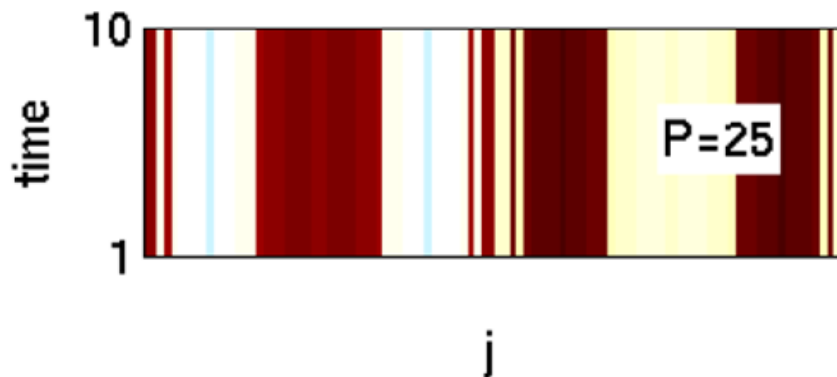
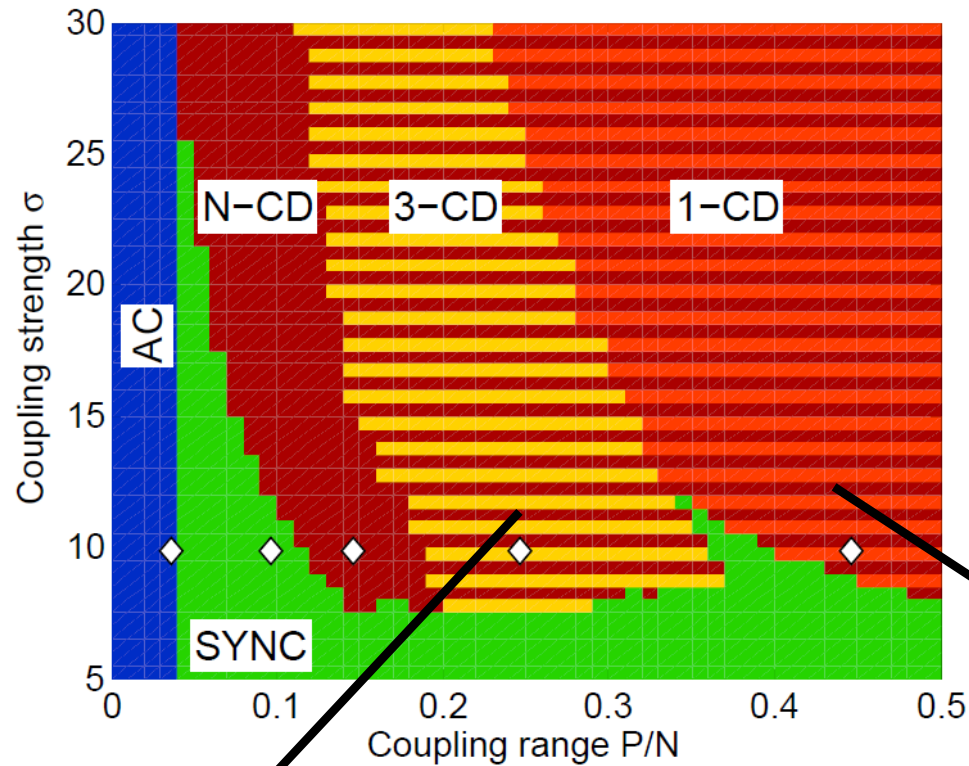


Chimera death: formation of clusters



What is the impact of the interaction range?

Chimera death



Chimeras in nature?

Unihemispheric sleep: one half of the brain is highly synchronized (sleeping) while the other half remains desynchronized (awake).

- **birds:** sleeping during the flight



Rattenborg, N.C., Voirin, B., Cruz, S. M., Tisdale, R., Dell'Omo, G., Lipp, H.-P., Wikelski, M. and Vyssotski, A. L., Evidence that birds sleep in mid-flight, Nature Comm. 7, 12468 (2016)

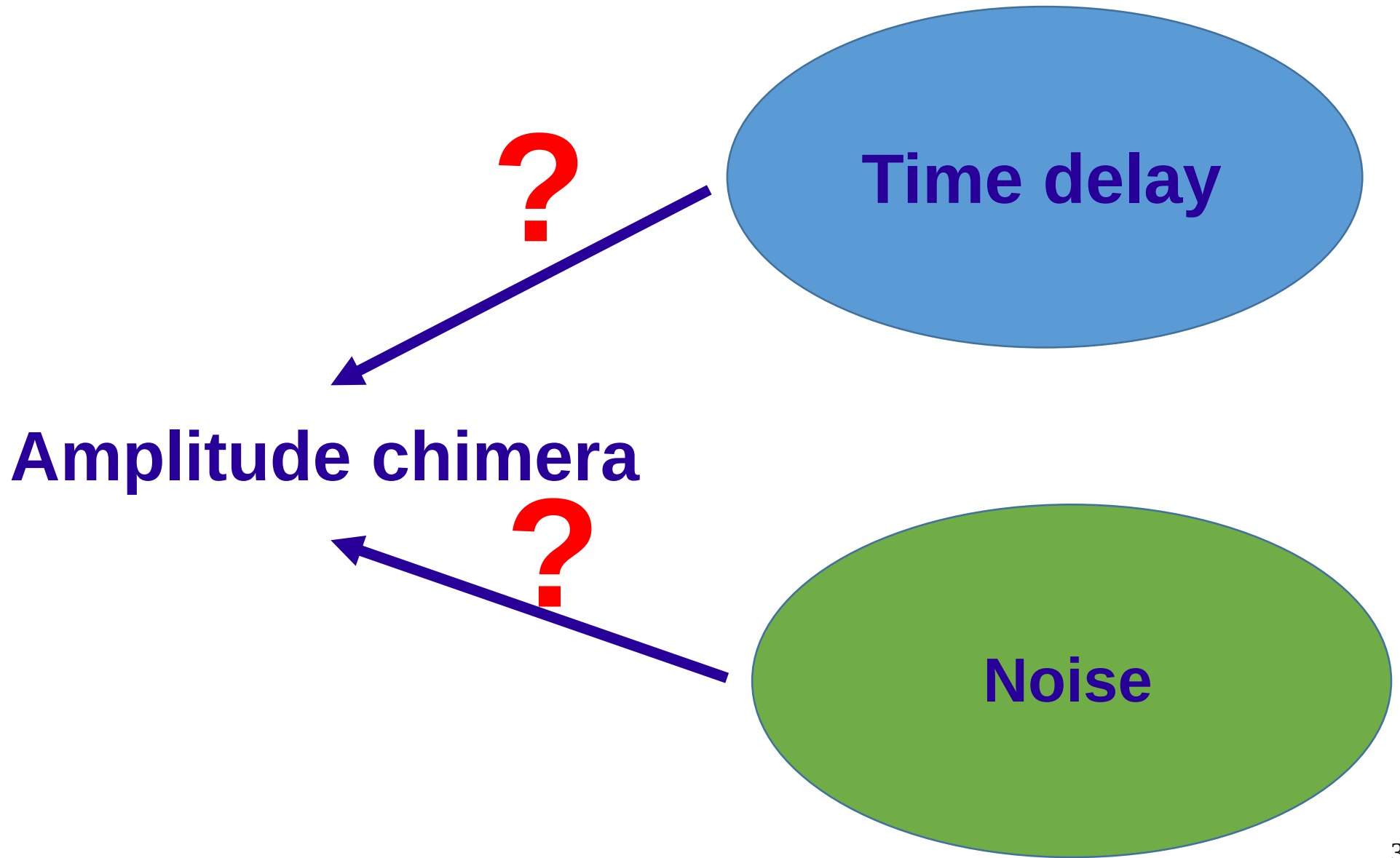
Chimeras in nature?

- **humans:** first-night effect – our brain stays alert to protect against unknown danger.



M. Tamaki, J. W. Bang, T. Watanabe, Y. Sasaki, Night Watch in One Brain Hemisphere during Sleep Associated with the First-Night Effect in Humans, *Curr Biol.* 26, 5 (2016)

Robustness of amplitude chimeras



Amplitude chimeras: impact of delay and noise

Network of Stuart-Landau oscillators

$$\dot{z}_j = f(z_j) + \frac{\sigma}{2P} \sum_{k=j-P}^{j+P} [Re(z_k) - Re(z_j)]$$

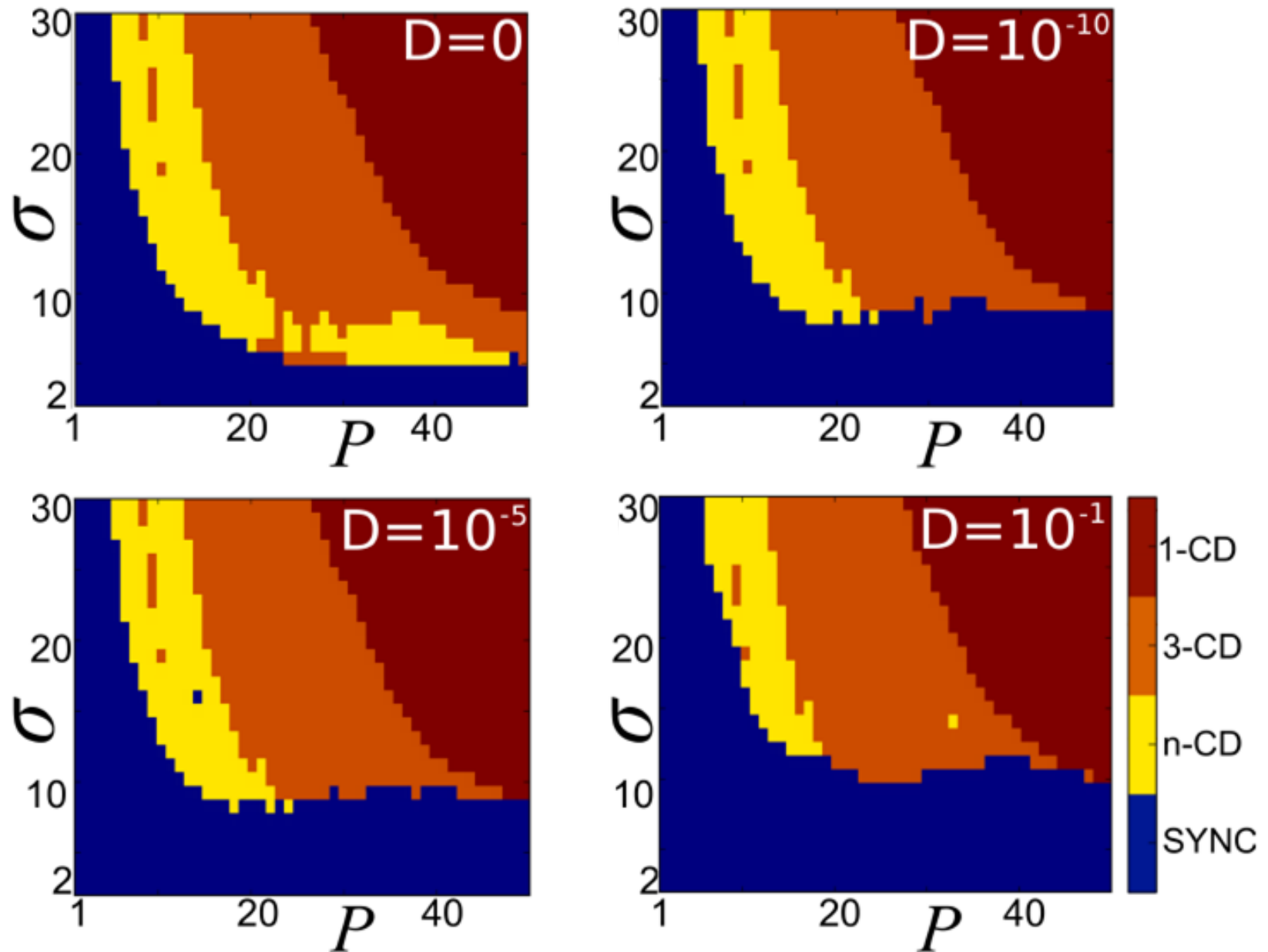
Network of Stuart-Landau oscillators with **delay**

$$\dot{z}_j = f(z_j) + \frac{\sigma}{2P} \sum_{k=j-P}^{j+P} [Re(z_k(t - \tau)) - Re(z_j(t))]$$

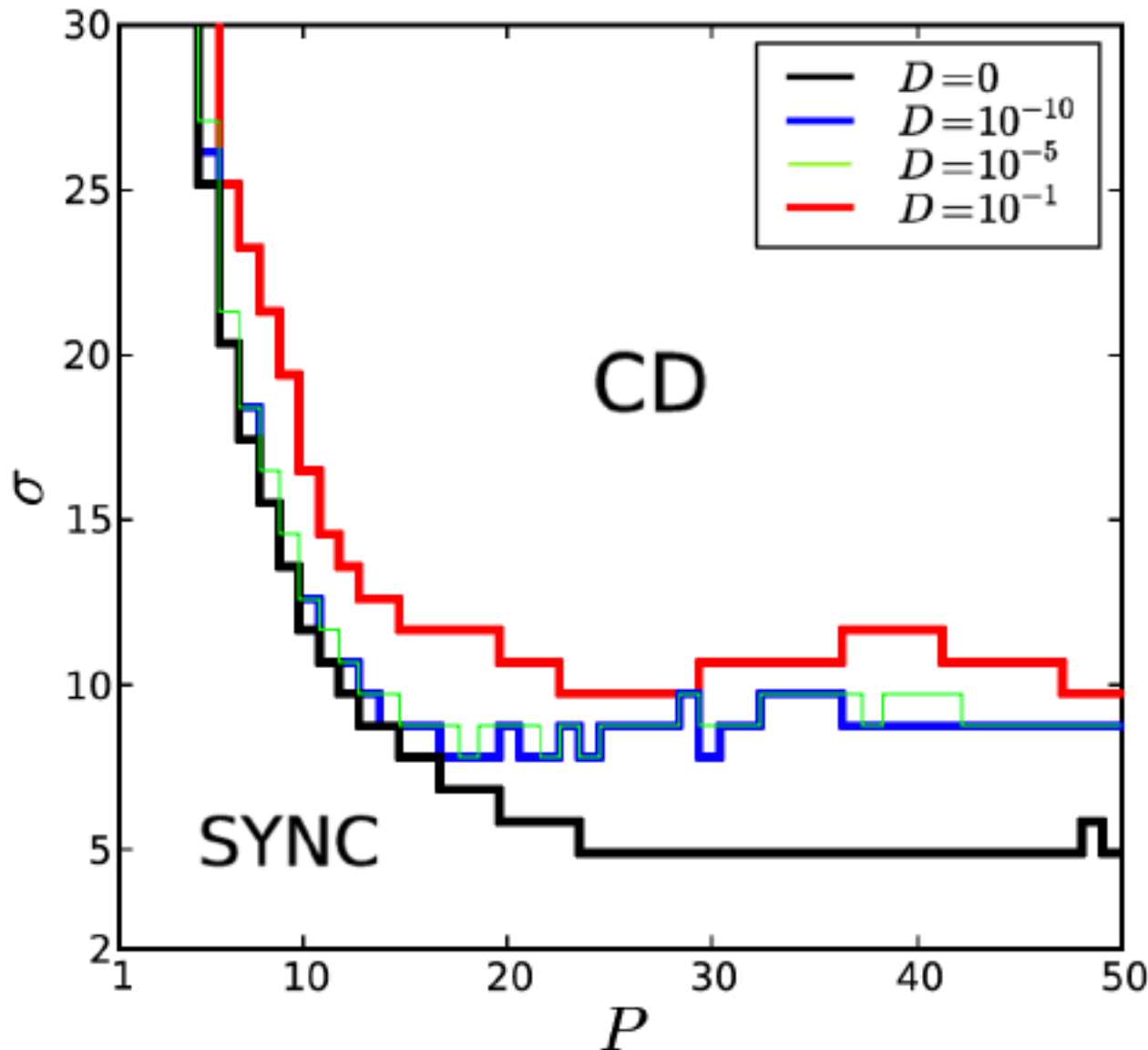
Network of Stuart-Landau oscillators with **noise**

$$\dot{z}_j = f(z_j) + \frac{\sigma}{2P} \sum_{k=j-P}^{j+P} [Re(z_k) - Re(z_j)] + \sqrt{2D} \xi_j(t)$$

Impact of noise

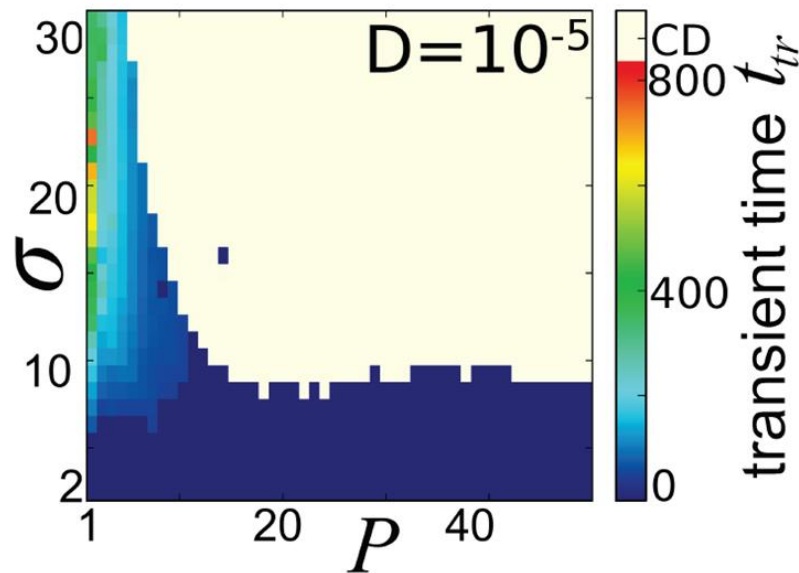
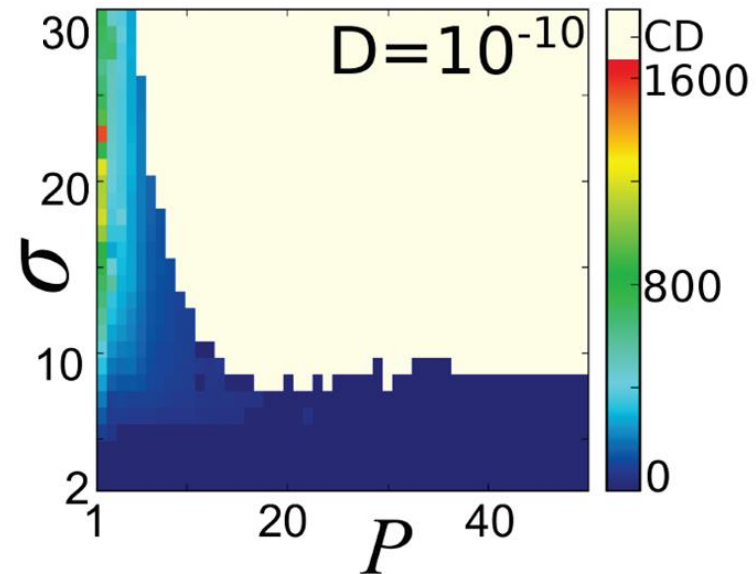
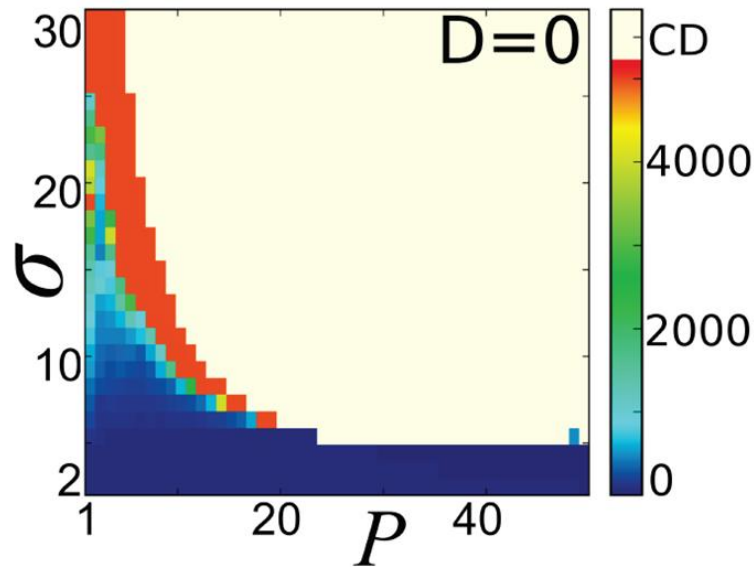


Impact of noise



**The stronger
the noise,
the larger the
oscillatory
regime (SYNC).**

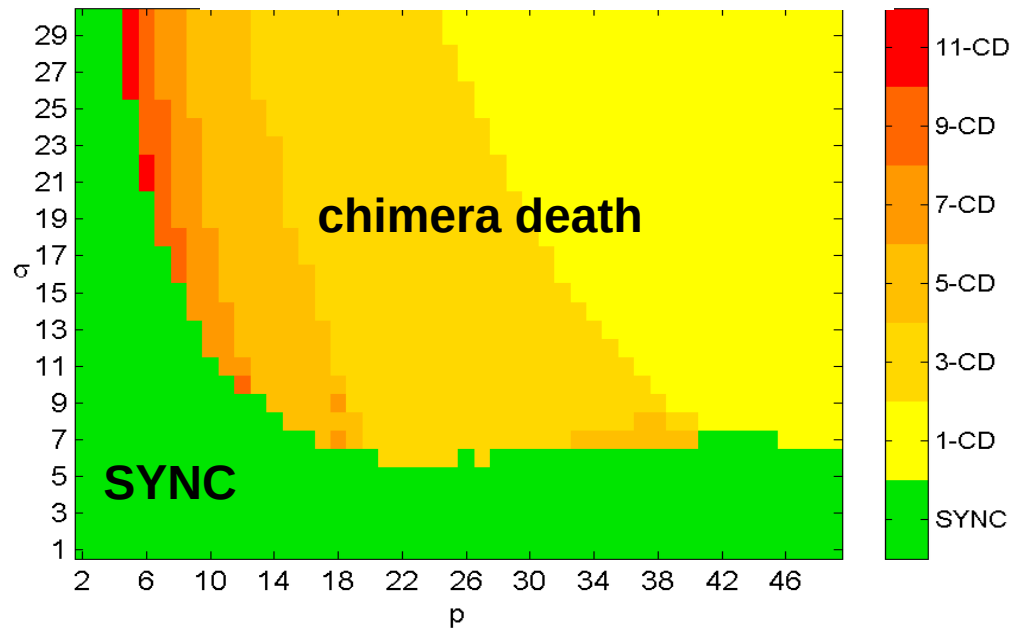
Impact of noise



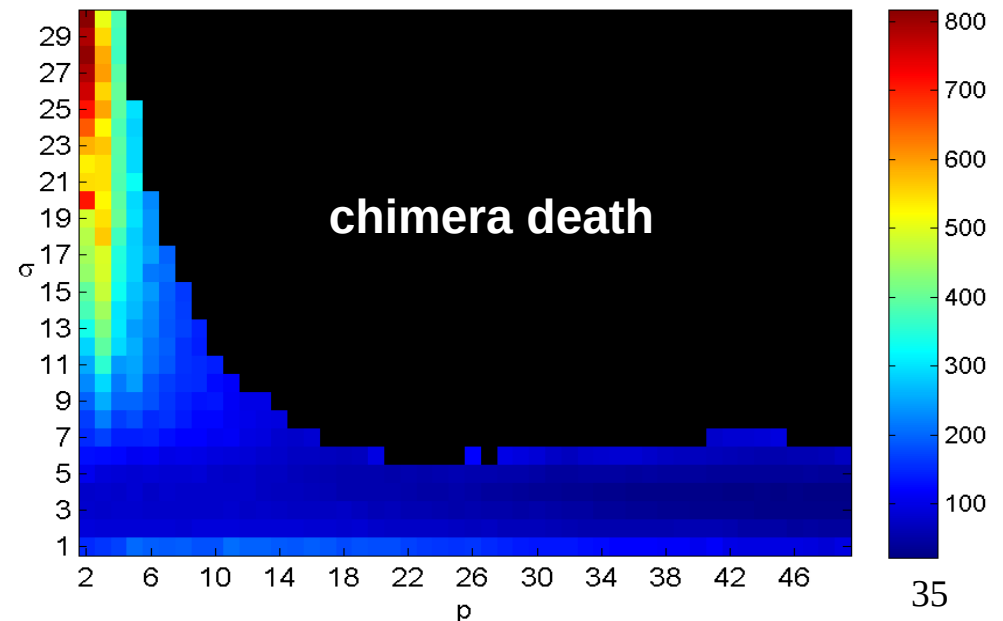
Noise decreases the lifetime of amplitude chimera.

Amplitude chimeras: no time delay

Map of regimes



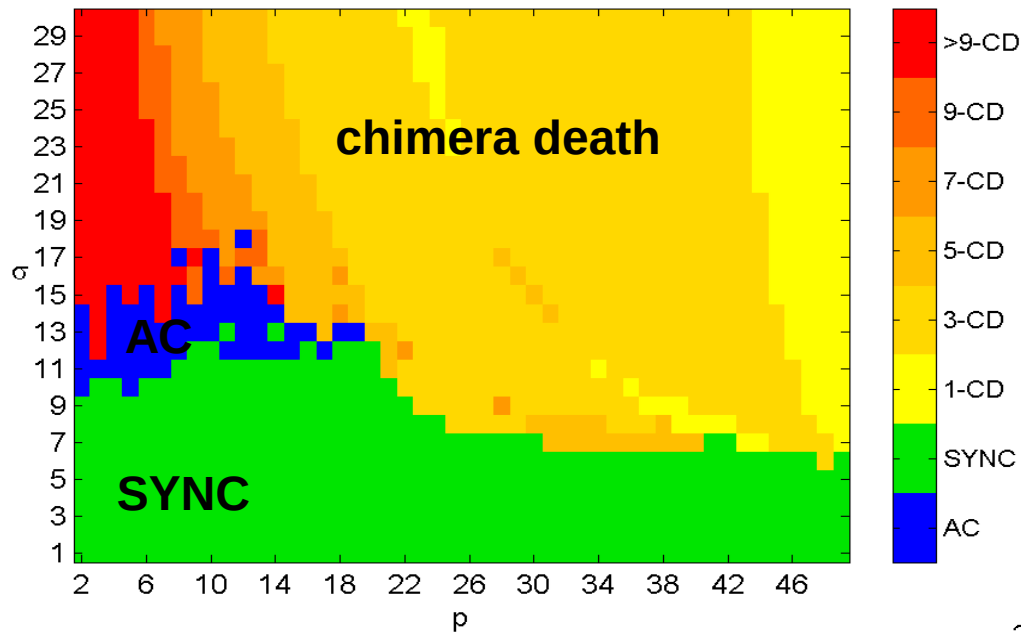
Lifetime of amplitude chimera



**No amplitude chimeras
beyond $t=900$!**

Impact of time delay ($\tau=\pi$)

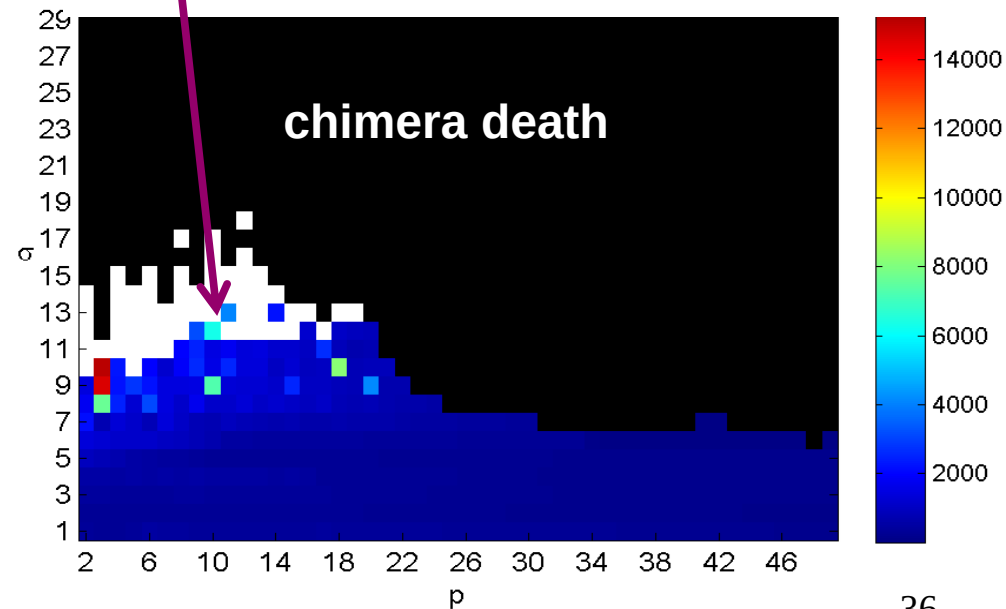
Map of regimes



**Time delay enlarges
the lifetime of
amplitude chimeras!**

Amplitude chimera lifetime $t=20000$

**Lifetime of amplitude
chimera**

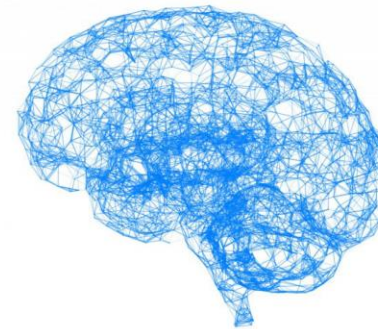


partial synchrony
in **multilayer** networks

Why multilayer?



Better representation of real-world systems



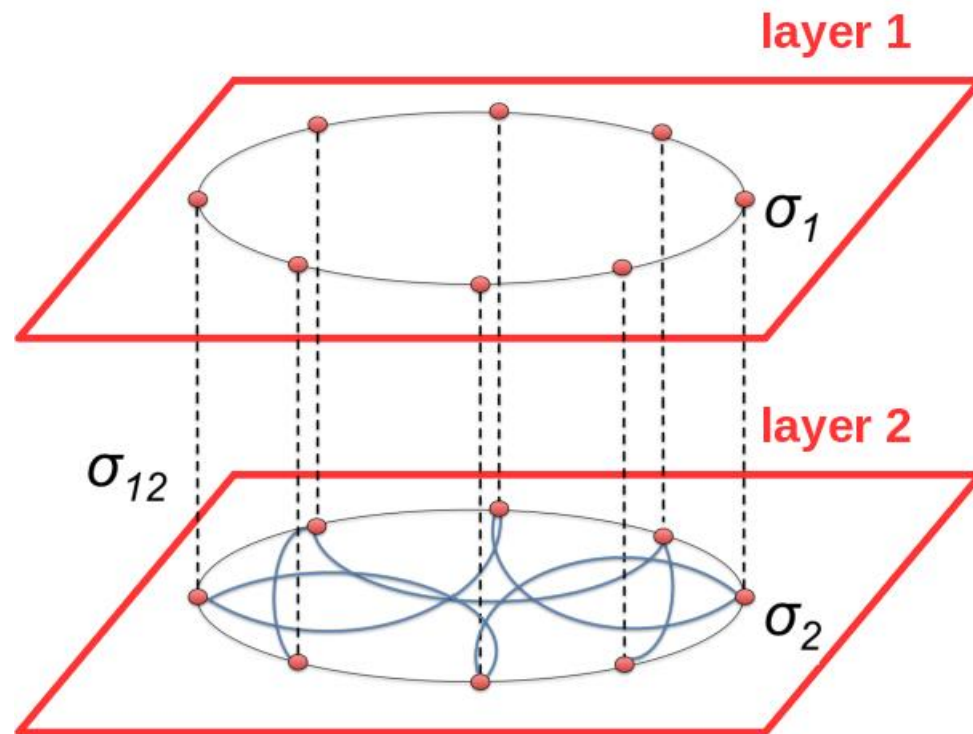
Reviews:

S. Boccaletti et al., The structure and dynamics of multilayer networks, Physics Reports 544, 1 (2014)

M. Kivelä, A. Arenas et al., Multilayer networks, Journal of Complex Networks 2, 3, 203 (2014)

What is a multilayer network?

A set of **nodes** interacting in **layers**,
each reflecting a distinct type of interaction.



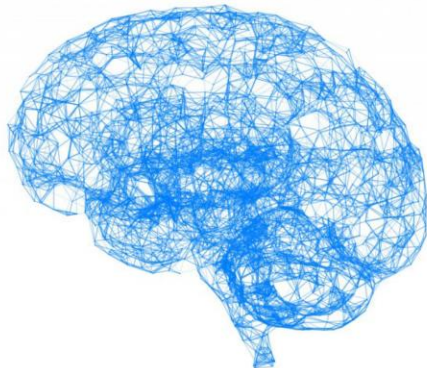
Examples

- **Social networks**: friendships in Facebook: family, friends, coworkers

The Facebook logo, consisting of the word "facebook" in white lowercase letters on a dark blue rectangular background.

- **Transportation networks**: air, train and bus transportation networks

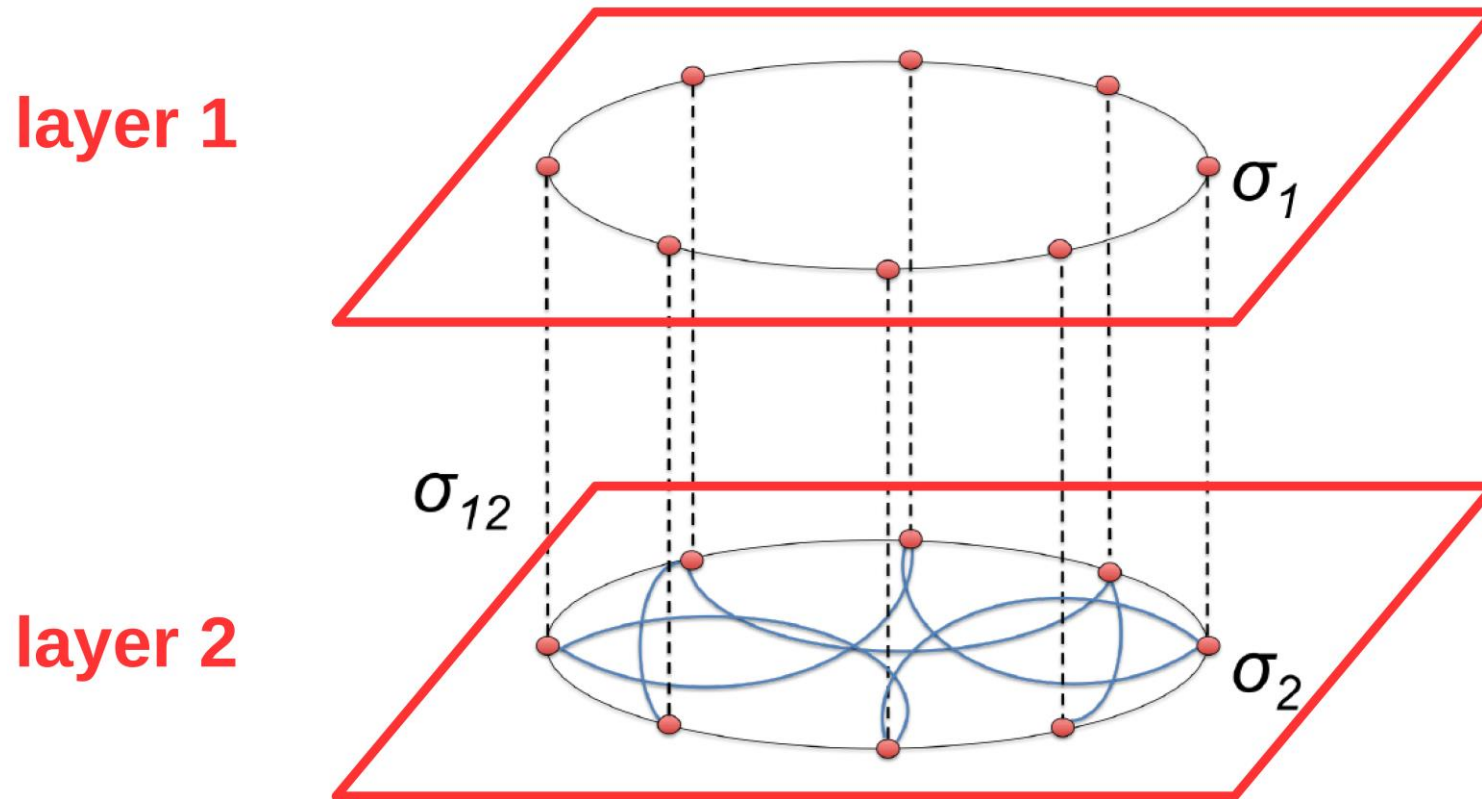
- **Neural networks**: chemical link or ionic channel



- **Brain networks**: different regions can be seen connected by functional and structural neural networks

Control

Control

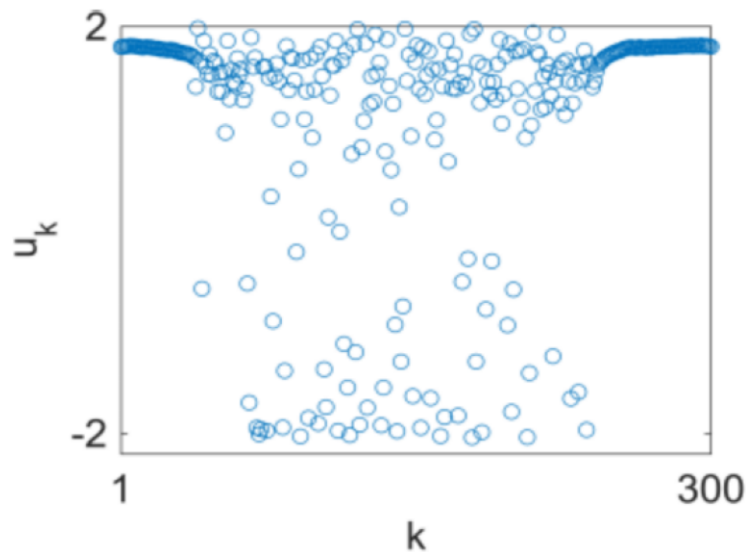


Can we control **one layer** by manipulating the parameters of **the other** layer?

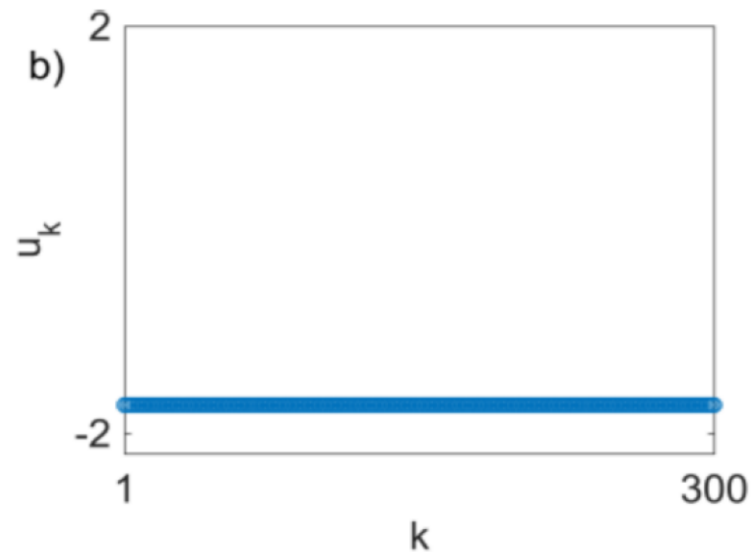
Control of chimera states

Isolated layers: different coupling strength

layer 1

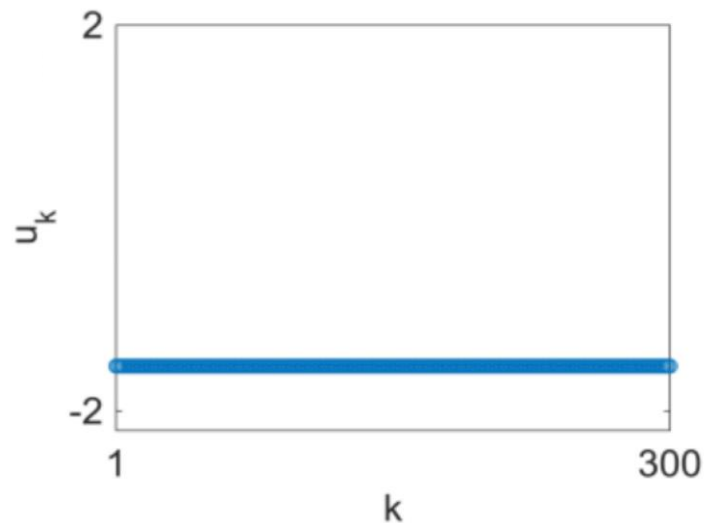


layer 2

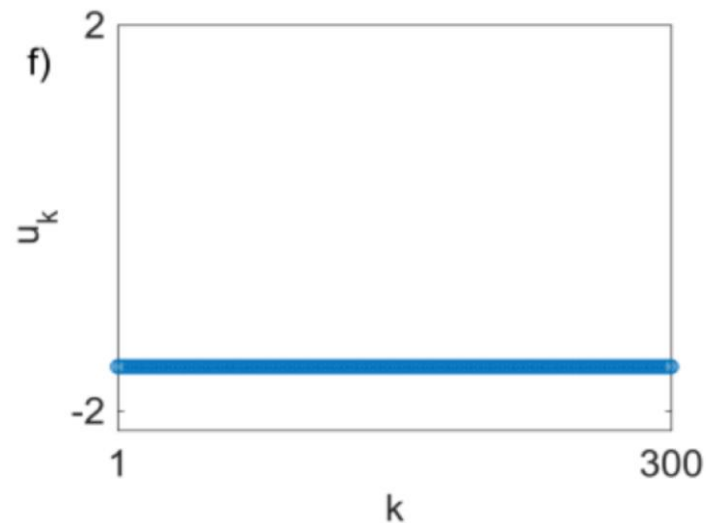


Coupled layers: chimera is **suppressed** in layer 1

layer 1



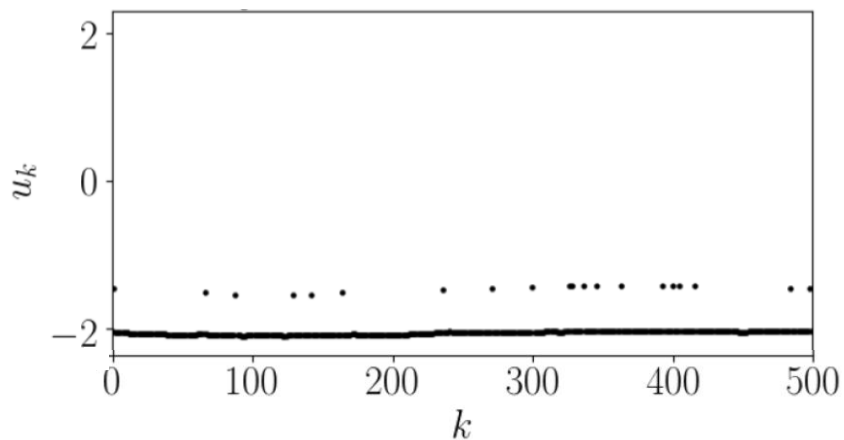
layer 2



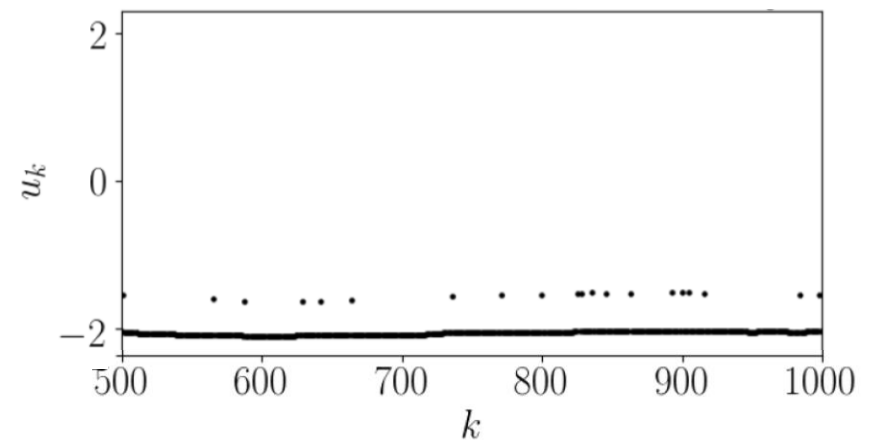
Control of solitary states

Coupled layers: solitary states are induced

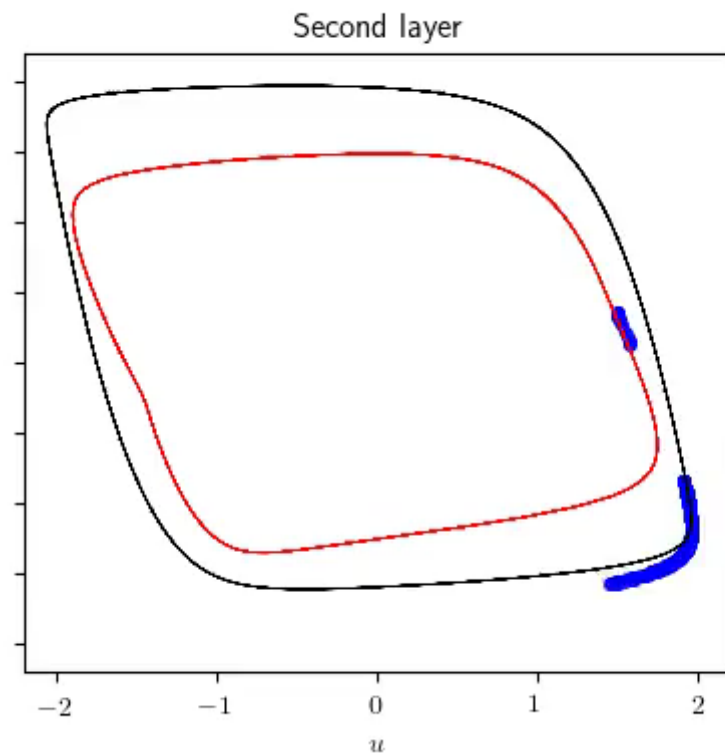
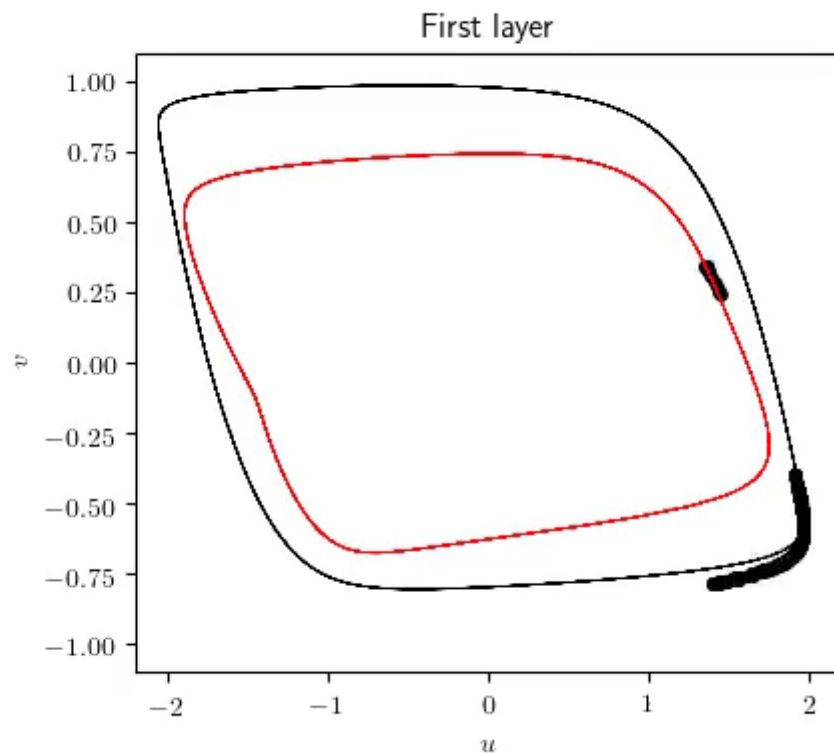
layer 1



layer 2



Coupled layers: solitary states are **induced**





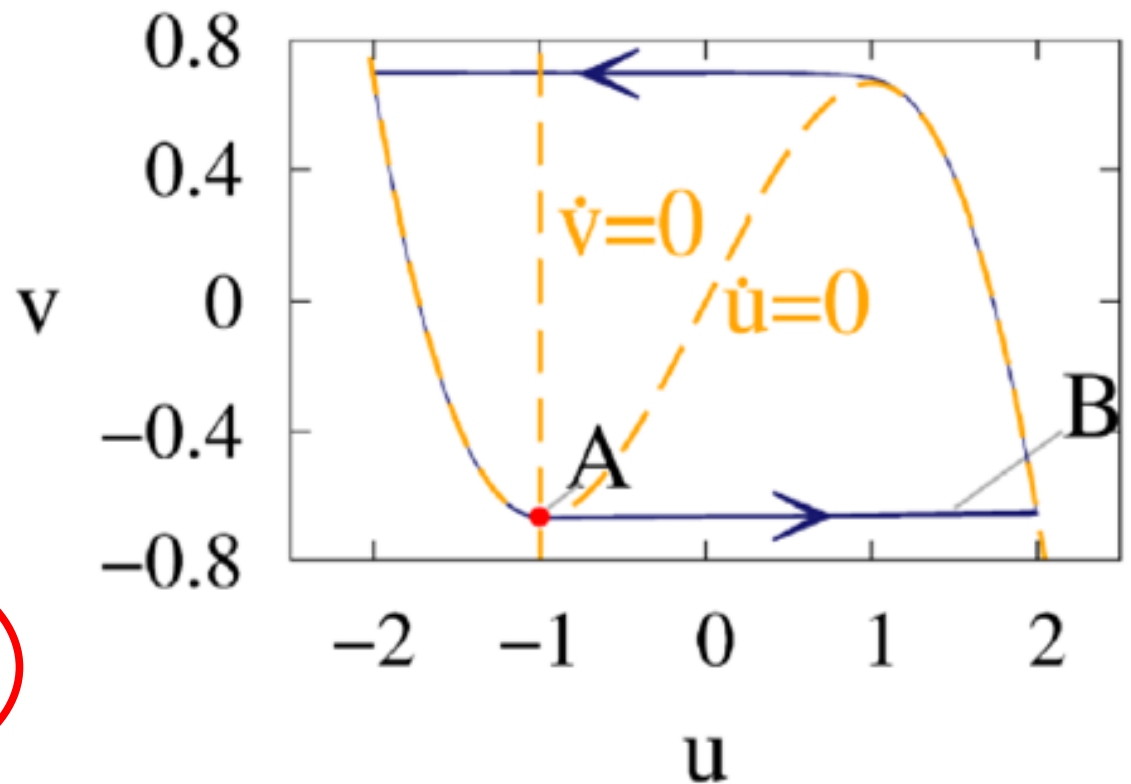
Constructive role
of noise?

Coherence-resonance chimeras

Model: FitzHugh-Nagumo system in excitable regime

Single node dynamics

$$\begin{aligned}\varepsilon \dot{u} &= u - \frac{u^3}{3} - v, \\ \dot{v} &= u + a + \sqrt{2D}\xi(t)\end{aligned}$$



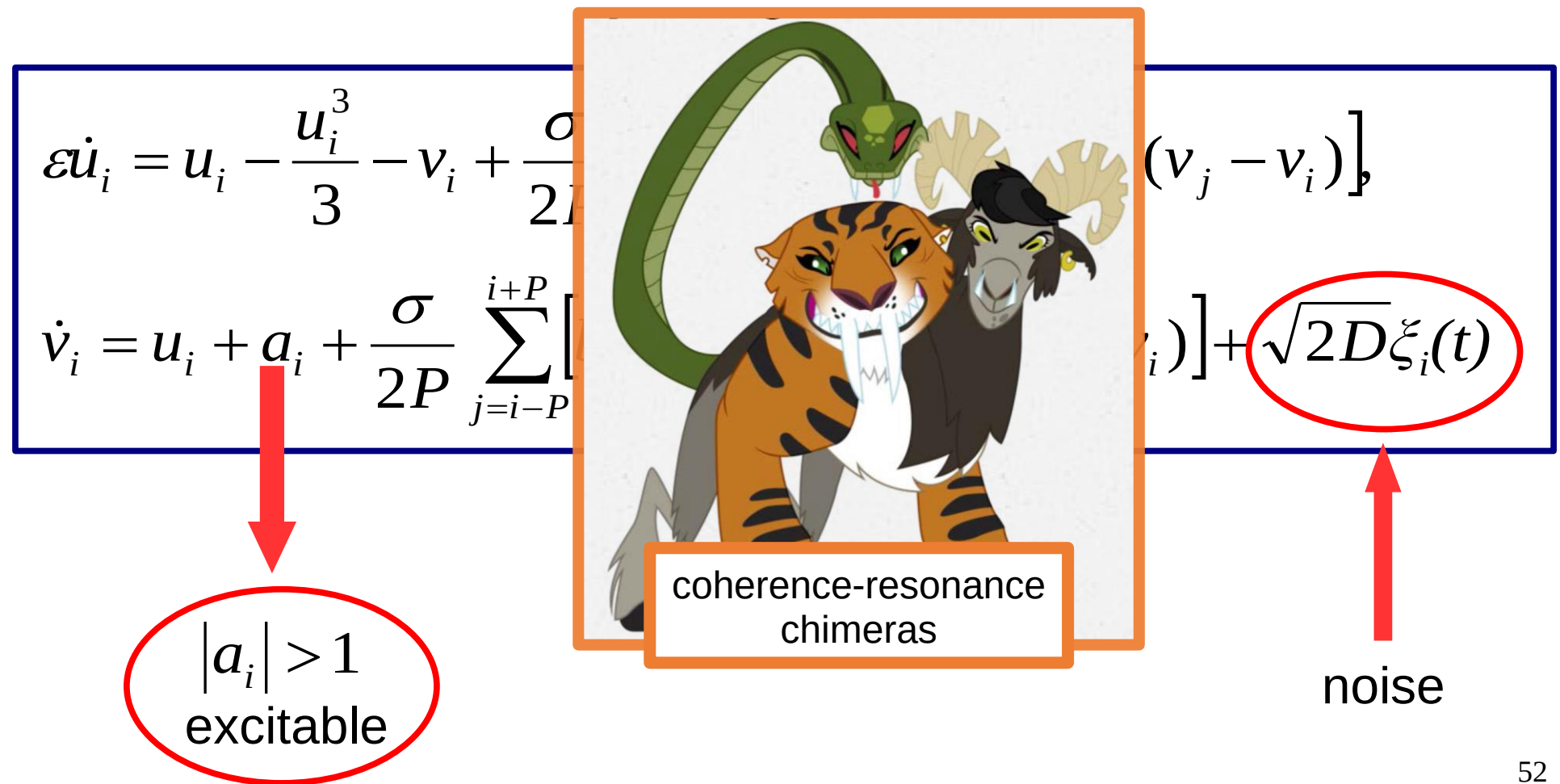
$|a_i| < 1$
oscillatory

$|a_i| > 1$
excitable

Model: network of **coupled** FitzHugh-Nagumo systems



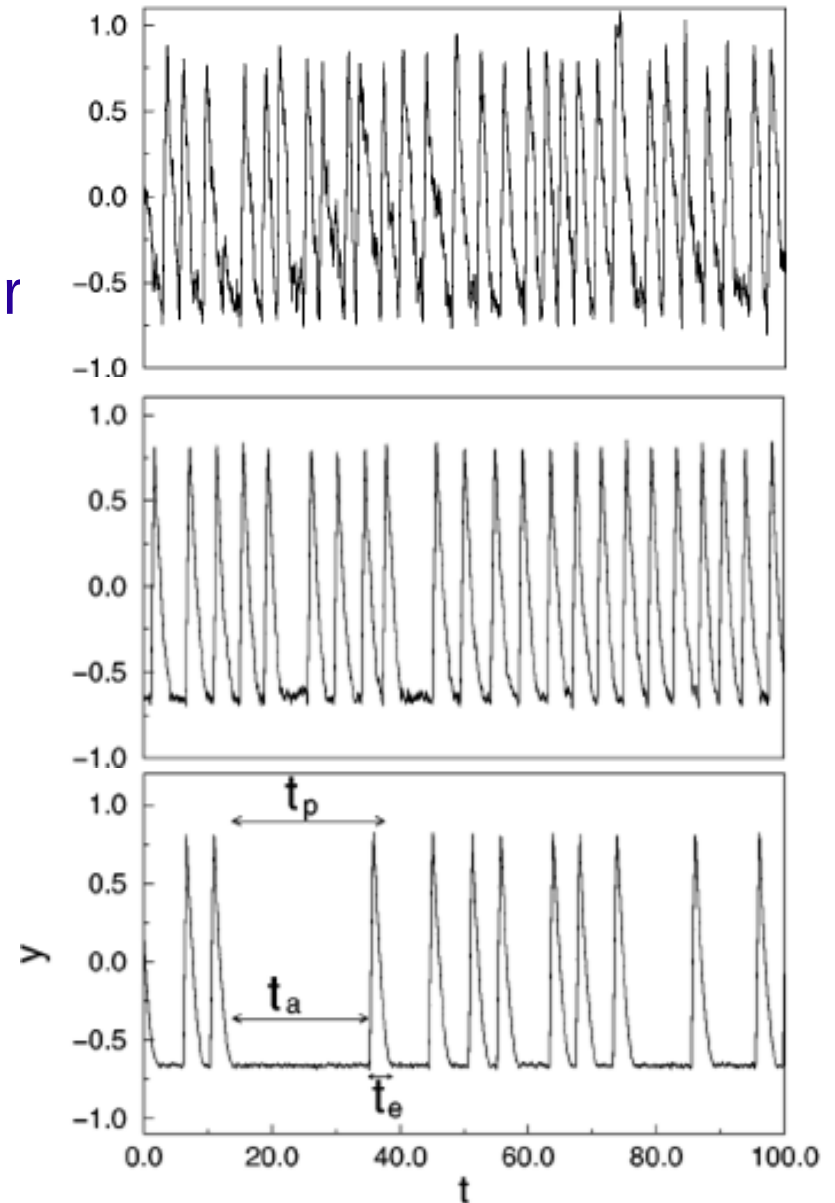
Nonlocally coupled ring



What is coherence resonance?

The best temporal regularity of the **noise-induced** oscillations occurs for an **intermediate** value of noise intensity

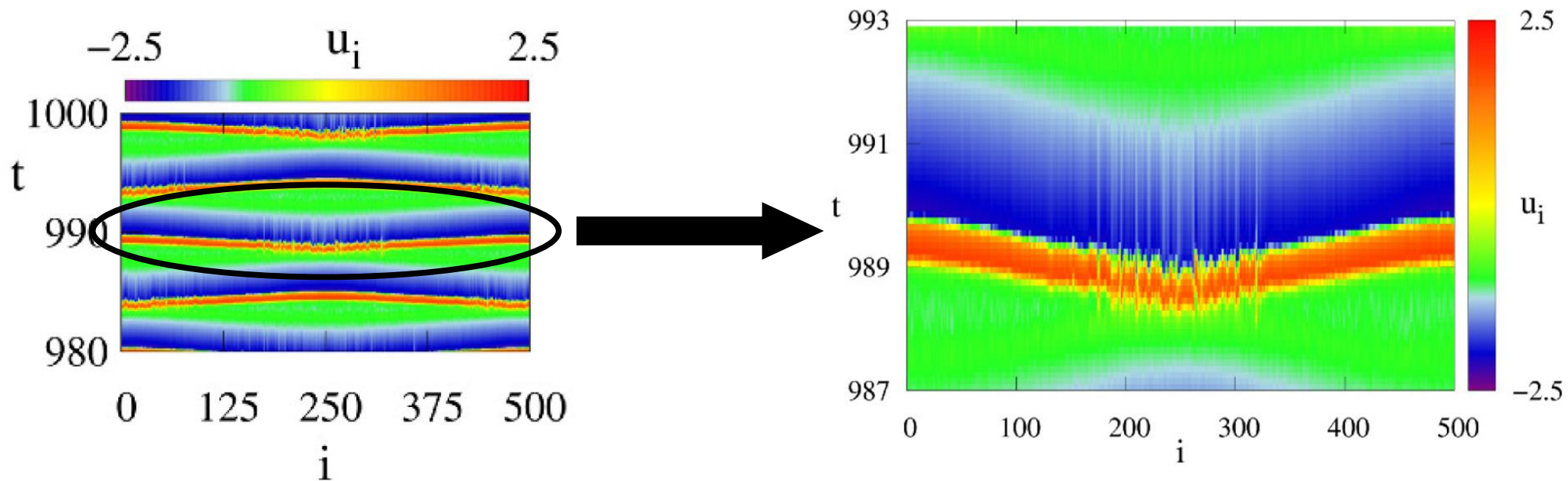
- discovered by Haken et al. in 1993
- named ***coherence resonance*** by Pikovsky and Kurths in 1997



constructive role of noise, **counter-intuitive** phenomenon

Coherence-resonance chimeras

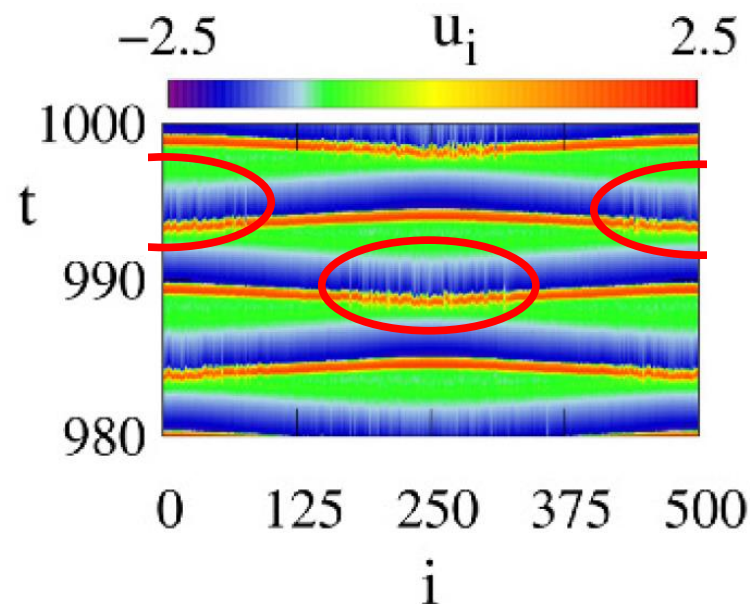
domains of **coherent** and **incoherent** spiking



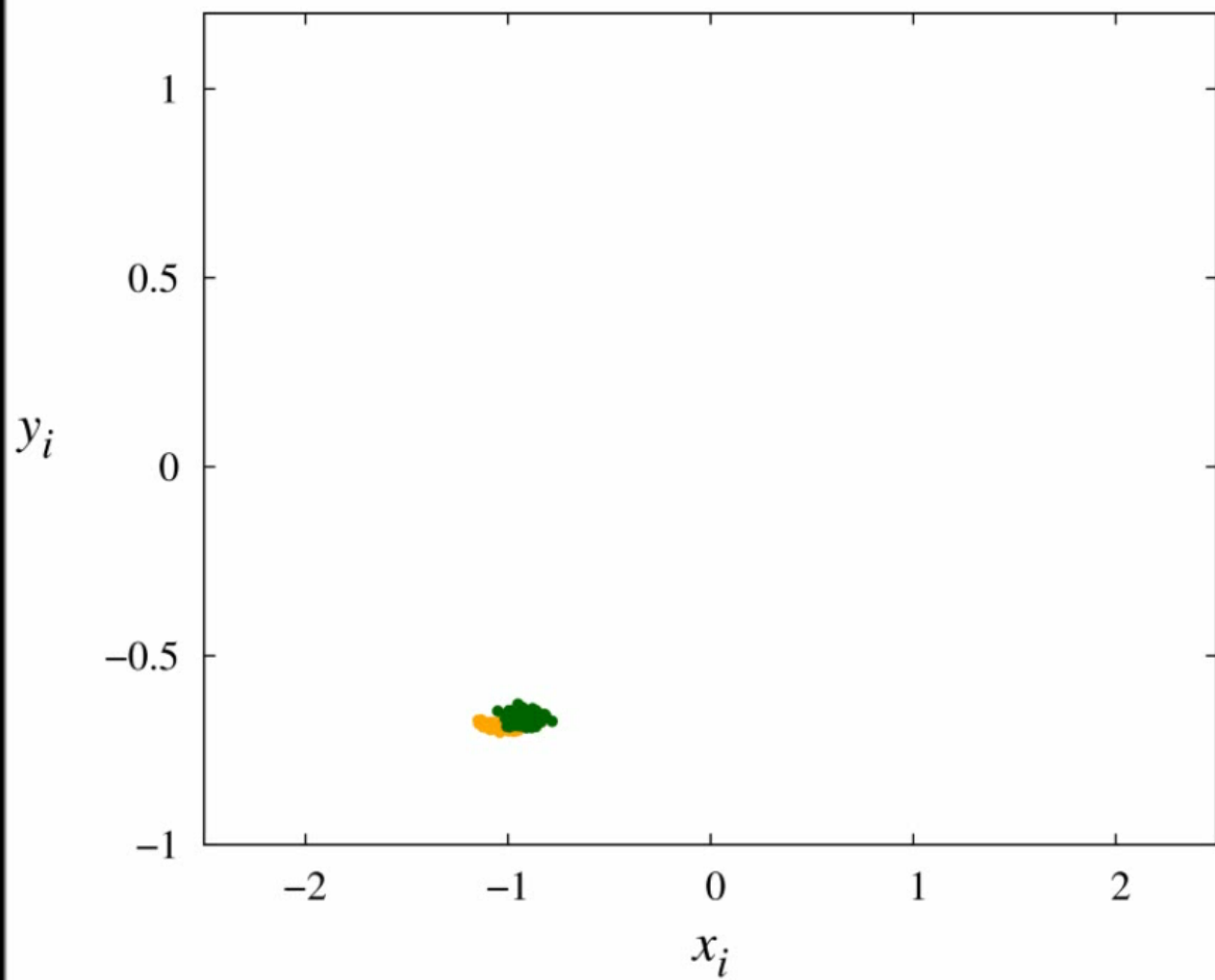
Intermediate noise intensities $0.000062 \leq D \leq 0.000325$!

Coherence-resonance chimeras

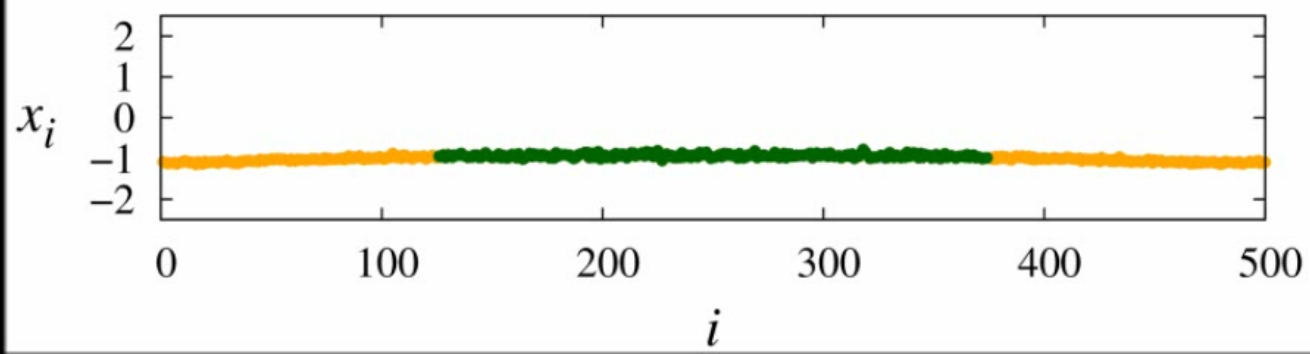
- noise-induced
- coherence resonance + chimera state
- alternating behavior



$\varepsilon=0.05$ $a=1.001$ $\sigma=0.400$ $r=0.200$ $D=0.0002$

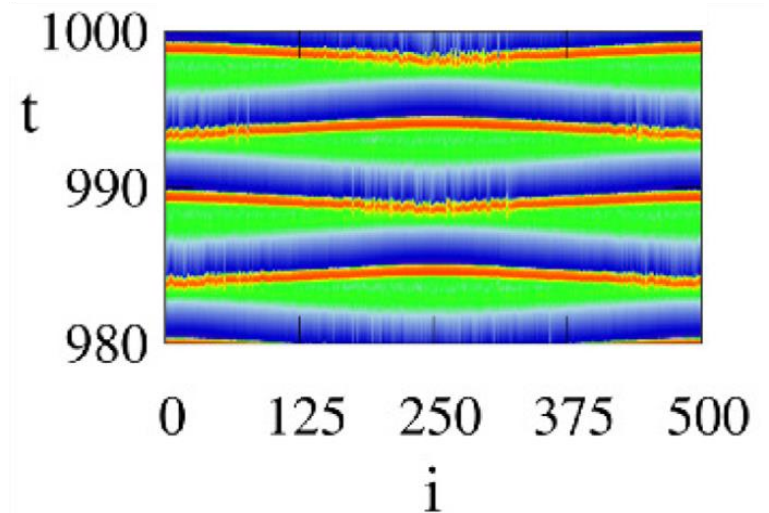
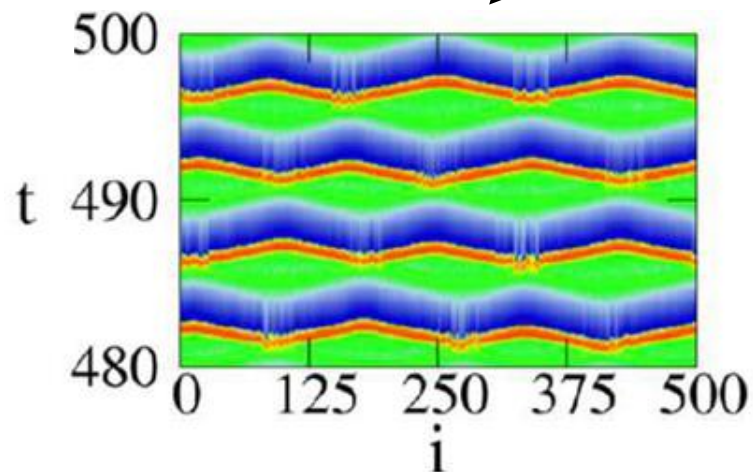
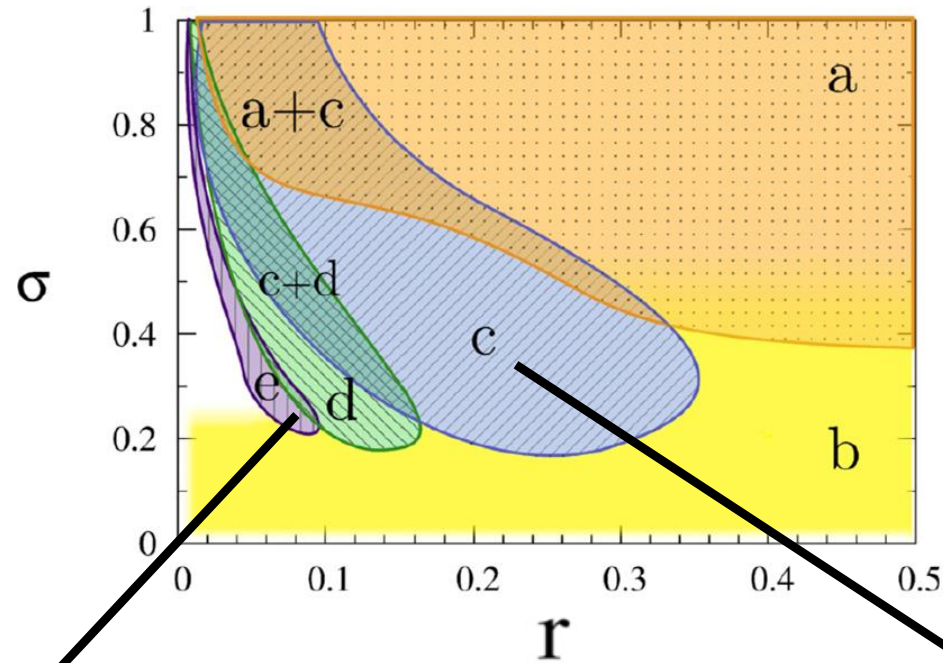


$t=976.89$



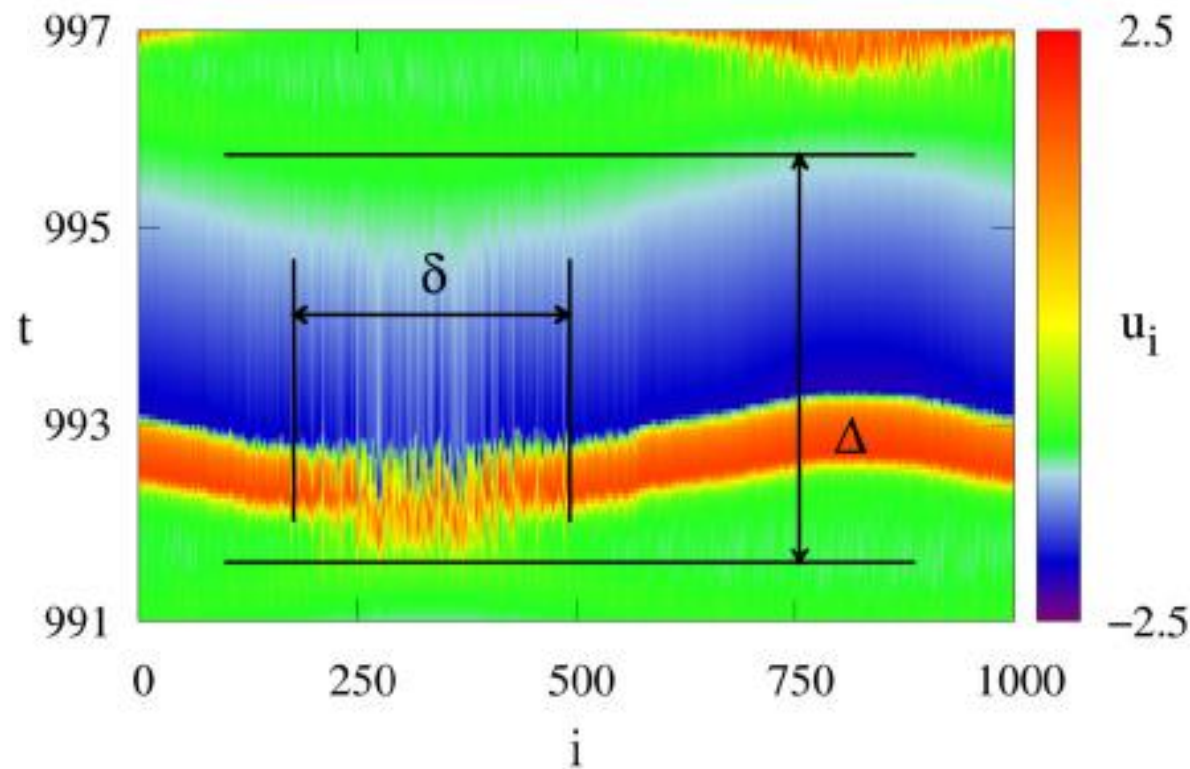
What is the impact of the interaction range?

Coherence-resonance chimeras



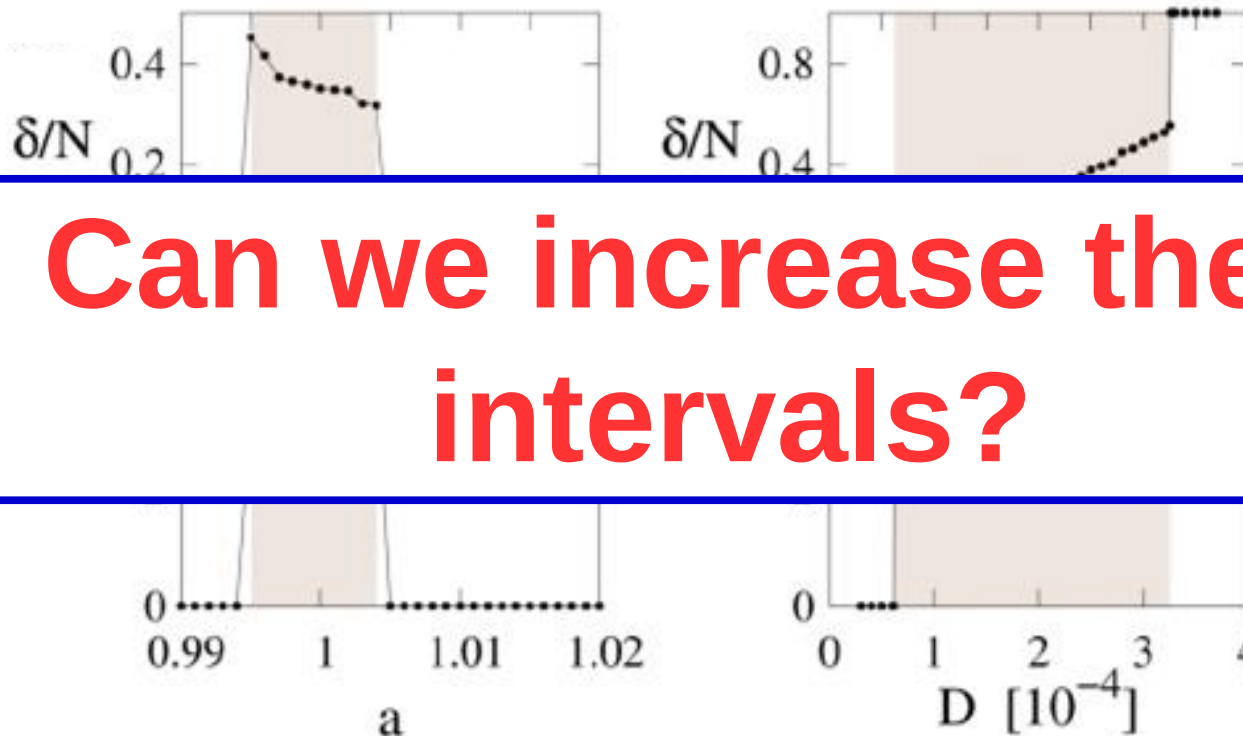
Coherence resonance chimera:

- active time span
- size of the incoherent domain



Coherence resonance chimera: interval of existence

- Threshold parameter a
- Noise intensity D



**Can we increase these
intervals?**

Time-delayed feedback

Model: network of coupled FitzHugh-Nagumo systems

Time-delayed feedback

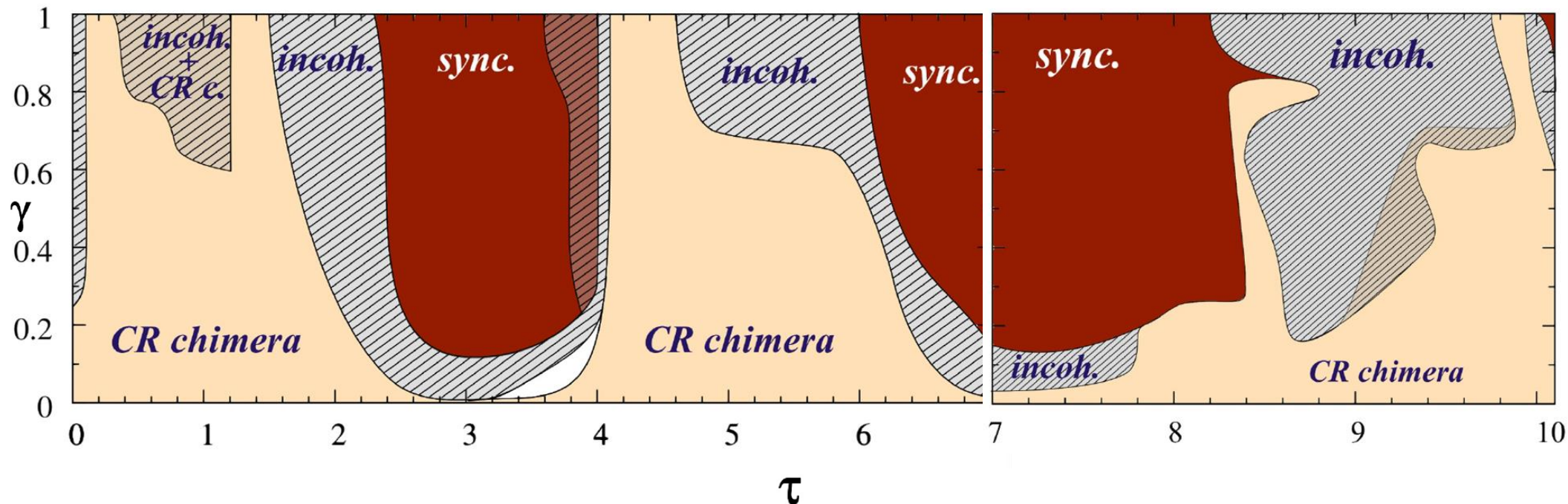
$$\begin{aligned}\varepsilon \dot{u}_i &= u_i - \frac{u_i^3}{3} - v_i + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} [b_{uu}(u_j - u_i) + b_{uv}(v_j - v_i)] + \gamma(u_i(t) - u_i(t - \tau)), \\ \dot{v}_i &= u_i + a_i + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} [b_{vu}(u_j - u_i) + b_{vv}(v_j - v_i)] + \sqrt{2D}\xi_i(t)\end{aligned}$$

feedback

Parameters -> coherence resonance chimera

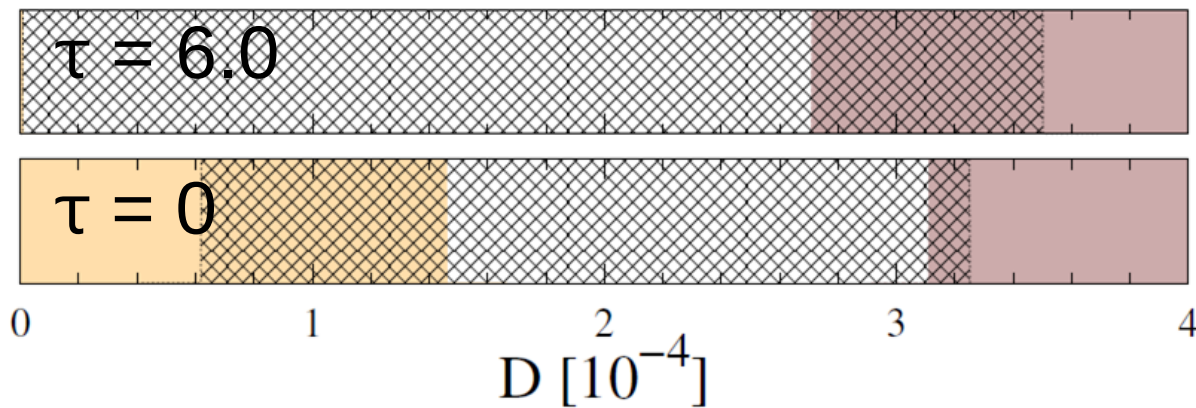
Map of regimes

- Feedback: strength and time delay
- “Synchronization tongues”



Impact of time delay

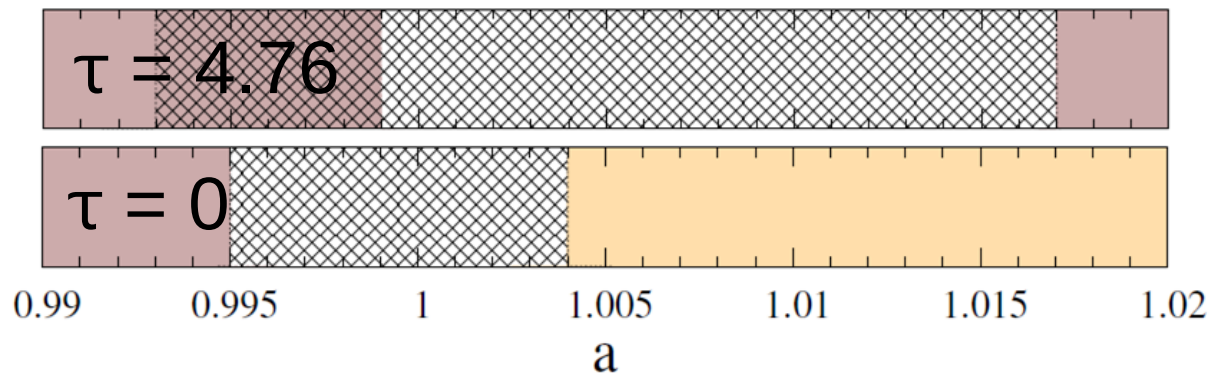
Noise intensity interval



Enlargement of the interval: 33%

Impact of time delay

Threshold parameter interval

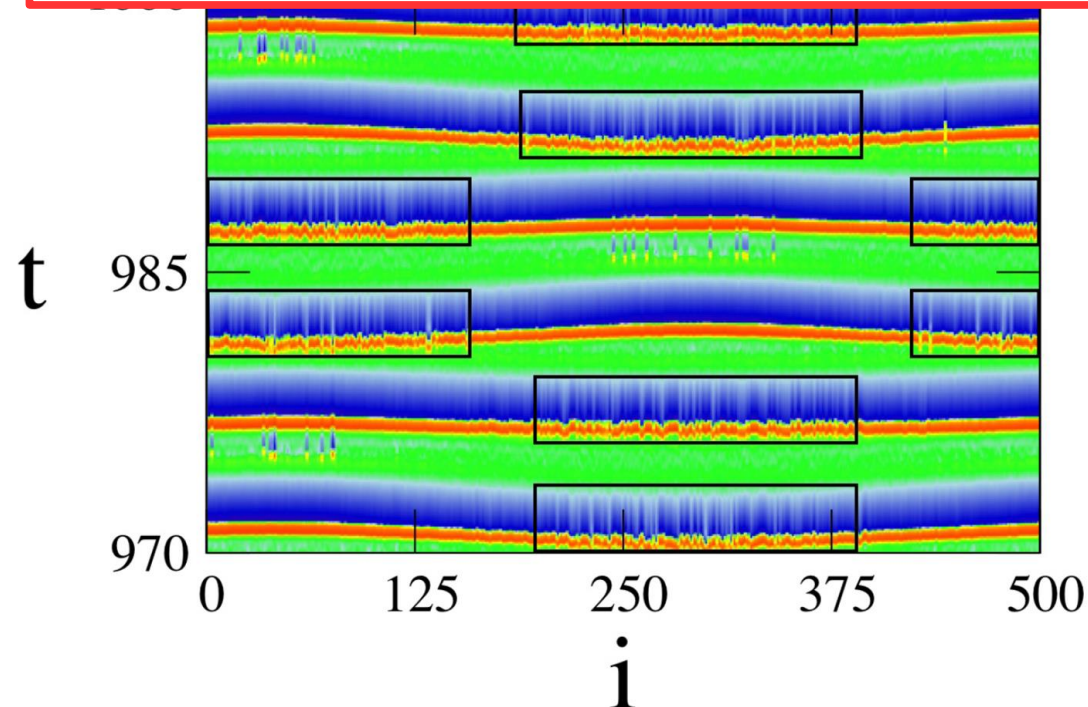


Enlargement of the interval: more than doubled

Delay-induced chimeras

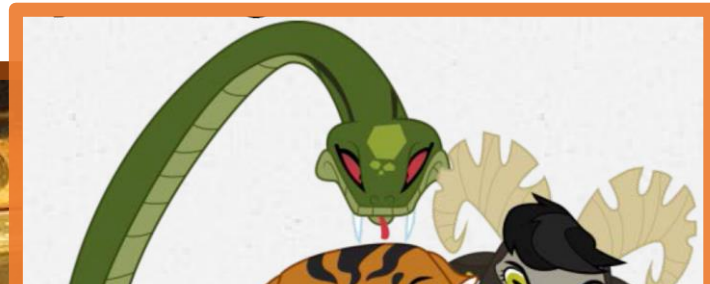
- Switching for every **second** spiking event

**Period-two coherence
resonance chimeras**

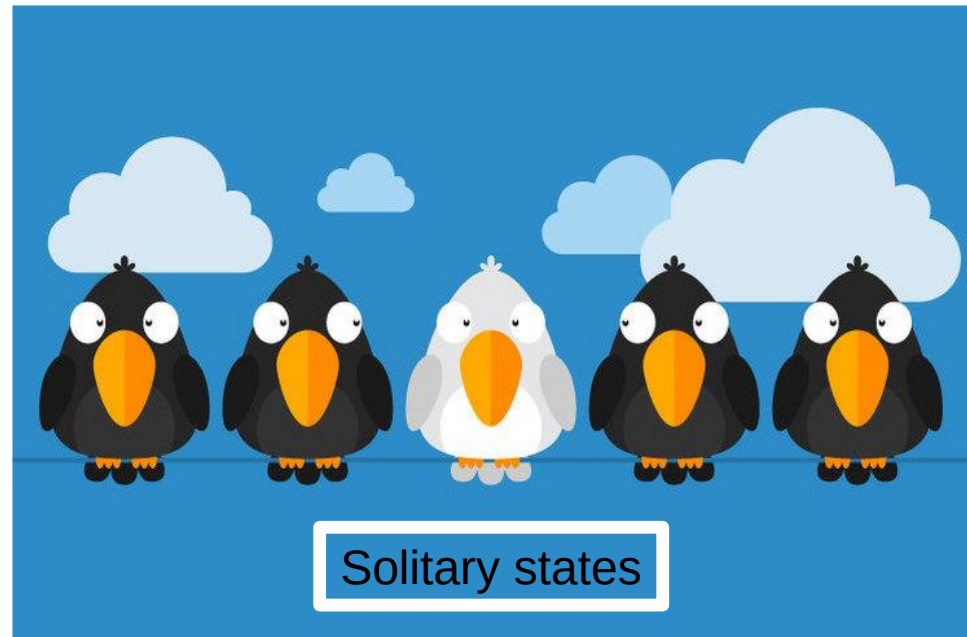




ampl



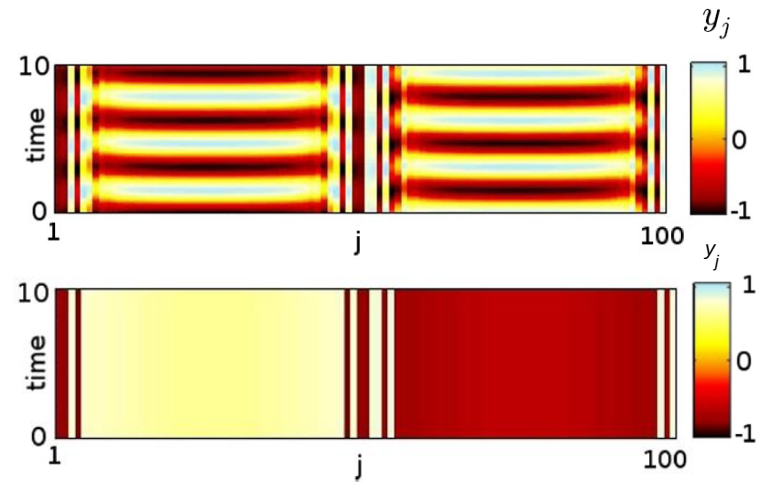
chimera death



Solitary states

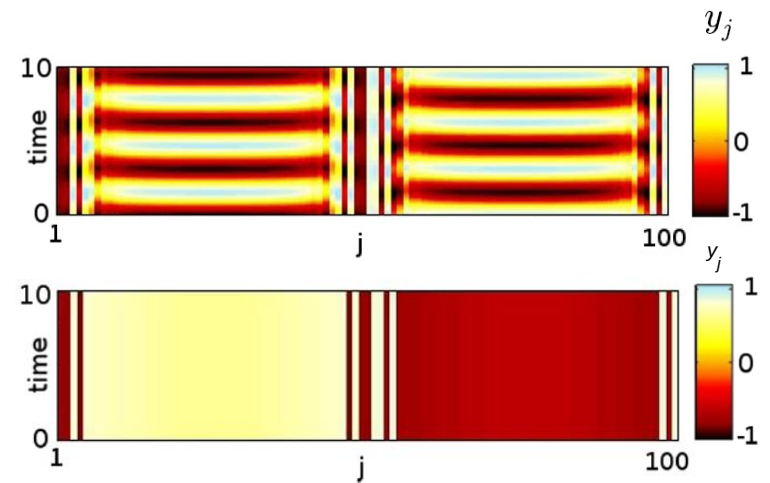
Different types of chimeras

Amplitude chimeras and
chimera death in ring networks
(Stuart-Landau oscillator)

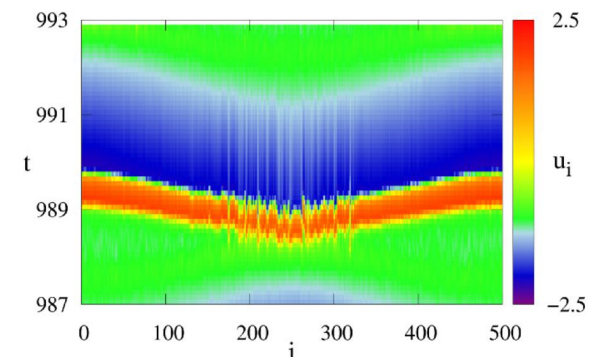


Different types of chimeras

Amplitude chimeras and
chimera death in ring networks
(Stuart-Landau oscillator)



Noise-induced chimera states –
coherence-resonance chimeras
(FitzHugh-Nagumo system)



Common trend:
decreasing coupling range $\rightarrow$ formation of clusters

What is the impact of delay and noise?

- Time delay and noise can induce **new dynamical regimes** (coherence-resonance chimeras, delay-induced period-two chimeras)
- **Time delay** allows to **control** chimera states (delay increases chimera lifetime)
- **Noise** can have both **destructive** and **constructive** role

