PD Dr. Gernot Schaller
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10. Übungsblatt – TPVI: Theorie des Quantentransportes

Abgabe: Do. 22.01.2020 16:00 Uhr im Tutorium
Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen oder Dreiergruppen erfolgen.

Aufgabe 20 (11 Punkte): *Second law*

Consider a fermionic exactly solvable model with two terminals where the matter and energy current from left to right are given by the Landauer formula

$$I_M = \frac{1}{2\pi}\int d\omega\, T(\omega)\left[f_L(\omega) - f_R(\omega)\right], \tag{1}$$

$$I_E = \frac{1}{2\pi}\int d\omega\, \omega T(\omega)\left[f_L(\omega) - f_R(\omega)\right], \tag{2}$$

where $T(\omega)$ is known as the transmission and $0 \leq T(\omega) \leq 1$. The entropy production rate is given by

$$\dot{S}_i = (\beta_R - \beta_L)\, I_E + (\beta_L\mu_L - \beta_R\mu_R)\, I_M. \tag{3}$$

(a) (2 Punkte) Using the Landauer formulas write the entropy production rate as $\dot{S}_i = \int d\omega\, T(\omega)\chi(\omega)$.

(b) (4 Punkte) Find the minimum of function $\chi(\omega)$.

(c) (5 Punkte) By looking at the concavity (second derivative) of $\chi(\omega)$ show that $\chi(\omega) \geq 0$ and that the second law ($\dot{S}_i \geq 0$) is obeyed.

Aufgabe 21 (7 Punkte): *Bogoliubov transformation*
Consider the following Hamiltonian

$$H = \omega(a^\dagger a + \frac{1}{2}) + \frac{\Delta}{2}(a^\dagger a^\dagger + aa), \tag{4}$$

where $a^\dagger$ and a are creation and annihilation bosonic operators. In order to diagonalize the Hamiltonian use the Bogoliubov transformation

$$b = ua + va^\dagger \tag{5}$$

$$b^\dagger = ua^\dagger + va, \tag{6}$$

with u and v real.

(a) (2 Punkte) Find the condition on u and v such that the operators b and $b^\dagger$ obey the canonical commutation relations.

(b) (5 Punkte) Find $\tilde{\omega}, u$ and v such that the transformation diagonalizes the Hamiltonian as $H = \tilde{\omega}(b^\dagger b + \frac{1}{2})$.

Aufgabe 22 (12 Punkte): *Reaction coordinate mapping*
Consider a quantum dot with energy ϵ_0 coupled to an electronic reservoir characterized by a spectral density of the form

$$\Gamma^{(0)}(\omega) = \Gamma\frac{\delta^2}{(\omega-\epsilon)^2 + \delta^2}. \tag{7}$$

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(a) (6 Punkte) For the parameters $\Gamma = 1\epsilon_0$, $\delta = 0.1\epsilon_0$ and $\epsilon = 1.5\epsilon_0$, find the residual coupling λ, the energy of the reaction coordinate ε_{RC} and plot $\Gamma^{(0)}(\omega)$ and the residual spectral density $\Gamma^{(1)}(\omega)$.

(b) (6 Punkte) Repeat (a) for a spectral density of the form

$$\Gamma^{(0)}(\omega) = \Gamma \frac{\delta^4}{\left[(\omega - \epsilon)^2 + \delta^2\right]^2}. \tag{8}$$

Vorlesung:	• Do. 10:00 Uhr – 12:00 Uhr im EW 203. • Fr. 10:00 Uhr – 12:00 Uhr im EW 203.
Übung:	• Do. 16:00 Uhr –18:00 Uhr im EW 733.
Scheinkriterien:	• Mindestens 60% der Übungspunkte. • Regelmäßige und aktive Teilnahme am Tutorium.