PD Dr. Gernot Schaller
Dr. Sebastian Restrepo

# 12. Übungsblatt – TPVI: Theorie des Quantentransportes

**Abgabe: Do. 06.02.2020 16:00 Uhr im Tutorium**

*Bei den schriftlichen Ausarbeitungen werden ausführliche Kommentare zum Vorgehen erwartet. Dafür gibt es auch Punkte! Die Abgabe soll in Zweiergruppen oder Dreiergruppen erfolgen.*

**Aufgabe 23 (30 Punkte):** *Coherent destruction of tunnelling*

The ability of a particle to tunnel between two regions of space is a purely quantum phenomenon that can be suppressed by means of periodic driving. This idea was explored in Grossmann, Dittrich, Jung, Hänggi *Phys. Rev. Lett.* **67**, 516 (1991). Representing the two regions of space by means of a two-level system, the model can be described by the Hamiltonian

$$H(t) = \Delta\sigma_x + g\cos(\Omega t)\sigma_z. \tag{1}$$

(a) (2 Punkte) Find the transformation $V(t)$ that removes the original time dependence from the Hamiltonian.

(b) (3 Punkte) Show that the Hamiltonian in the rotating frame has the form

$$H_{\text{rot}}(t) = \Delta f_x(t)\sigma_x + \Delta f_y(t)\sigma_y, \tag{2}$$

and find the form of the functions $f_x(t)$ and $f_y(t)$.

(c) (5 Punkte) In the high frequency regime $\Omega \to \infty$, an appropriate, time-independent approximation of the Hamiltonian $H_{\text{rot}}(t)$ involves averaging over the period,

$$H_{\text{rot}} \simeq \frac{1}{T}\int_t^{t+T} H_{\text{rot}}(\tau)d\tau, \tag{3}$$

where $T = \frac{2\pi}{\Omega}$ is the period of the Hamiltonian. In this limit, determine the values of $\frac{2g}{\Omega}$ that suppress the Hamiltonian.
*Hint:* The definition of the zeroth order Bessel function is

$$J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos\left[x\sin(\tau)\right] d\tau. \tag{4}$$

(d) (5 Punkte) If at time zero the system is in the localized state $|+\rangle$, where $\sigma_z|\pm\rangle = \pm|\pm\rangle$. Calculate the probability for the TLS to be in the same state $|+\rangle$ as a function of time? What happens to the probability at the values of $\frac{2g}{\Omega}$ found in (c)?

(e) (5 Punkte) Compare the previous result, with the high frequency limit taken without moving to a rotating frame. What is the advantage of moving to a rotating frame?

(f) (5 Punkte) Consider now the case where the system is coupled to a heat bath of inverse temperature $\beta$ and the interaction Hamiltonian is $H_I = \sigma_x \sum_k g_k(b_k + b_k^\dagger)$. Can coupling to the environment be suppressed in the high frequency limit?

(g) (5 Punkte) Consider now the case where the interaction Hamiltonian is $H_I = \sigma_z \sum_k g_k(b_k + b_k^\dagger)$. In the high frequency limit, what is the steady (long time limit $t \to \infty$) state of the system in the rotating frame?

| | |
|---|---|
| Vorlesung: | • Do. 10:00 Uhr – 12:00 Uhr im EW 203.<br>• Fr. 10:00 Uhr – 12:00 Uhr im EW 203. |
| Übung: | • Do. 16:00 Uhr –18:00 Uhr im EW 733. |
| Scheinkriterien: | • Mindestens 60% der Übungspunkte.<br>• Regelmäßige und aktive Teilnahme am Tutorium. |